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STOCHASTIC ANALYSIS OF UNCERTAINTY IN A U.S. INPUT-OUTPUT MODEL

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<u>ERRATA NOTICE:</u> Pages 32 - 36 were inadvertently incorrectly numbered. The continuity of the text is correct,

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1.0 INTRODUCTION

This report describes a stochastic parametric sensitivity analysis of a detailed structural model of the U.S. economic system. Model parameters defining the structure of the system at the time of measurement were derived from physical observations of the system. Use of such models is becoming increasingly prevalent for mid-to long-range studies and policy analyses in government planning at all levels. Resource scarcity, foreign policy contingencies and other factors have made rapid structural change the object of analysis, not something one can assume away. Effective use of such models requires an understanding of the effects of parametric change and uncertainty.

We are concerned here with a linear static input-output model of the U.S. economy. Its parameters are derived from data on interindustry transactions complied by the U.S. Department of Commerce. Due to the size and complexity of the economic system, funding limitations and measurement lags, these parameters are seven years out of date when published. Parametric uncertainty therefore can arise from two sources: observation of the system during the base year and structural changes during the seven year lag period. Estimates of uncertainty in the base year parameters were compiled by Bullard (1976) and are discussed in Appendix A.

The effect of parametric uncertainty on model outputs has been discussed by Sebald (1974) and Bullard and Sebald (1975). These papers quantified the maximum error tolerances that would result from the worst-case distribution of parametric errors. For this model, it was found that the process of matrix inversion could magnify input errors by more than 600%, emphasizing the need for developing a methodology that could quantify the extent to which parametric errors cancel one another.

The Monte Carlo simulation analysis described here was designed to answer that question. Base year interindustry transactions were characterized as random variables and the model parameters were derived from them. The results from each simulation were used to update a set of sufficient statistics to yield unbiased estimates of means, variances and some covariances. The simulations were performed to evaluate both the effect of doubling error tolerances on inputs and the effect of changing the structure of the model to enhance its usefulness for predictive work.

After 200 simulations, the preliminary results were analyzed in order to determine the cost-effectiveness of proceeding with more simulations. In all cases, this a priori determination of the confidence intervals on final results showed that 1000 runs would be adequate. These estimates were then verified when the simulation had been completed.

Chapter 2 describes the preparation of the data base and estimation of uncertainty on base year transactions. Chapter 3 details the simulation methodology, the criteria for determining "acceptability" of simulated parameters and derivation of the stopping rule. Chapter 4 presents the results of all simulations, and discusses the effects of aggregation, magnitude of input uncertainty and other variables.

2.0 DATA BASE PREPARATION

2.1 The Model

The linear static input-output model of the U. S. economic system is described in detail by Bullard and Herendeen (1975). It is based on the theory developed by Leontief (1941), and relies largely on data assembled by the U. S. Department of Commerce, Bureau of Economic Analysis (BEA). Data are expressed in constant dollars, which act as a surrogate for physical units. In this particular model however, the inputs of energy to all sectors are expressed in physical units, to take account of the fact that energy is sold to different users at different prices.

The governing equation of the model is

$$(I-A)\,X = Y \qquad (2.1\text{-}1)$$

where X is an N-order vector of gross domestic outputs for each sector, Y is the vector of total final demands for the output of each sector, and A is the matrix of parameters describing the technology of producing goods and services during the base year. A typical element A_{ij} represents the amount of input from sector i required directly by sector j to produce one unit of its output. These parameters are derived from base year observations of interindustry transactions, T_{ij}, (amount of output from sector i sold directly to sector j):

$$A_{ij} \equiv \frac{T_{ij}}{X_j} \qquad (2.1\text{-}2)$$

In turn, these interindustry transactions are defined as the sum

$$T = DA + MDT + TF \qquad (2.1\text{-}3)$$

where DA_{ij} is the amount of product i sold directly to sector j, MDT_{ij} represents the transportation or trade margin i on all inputs to sector j, and TF_{ij} represents the amount of product j produced as a secondary output by sector i.

2.2 The Data

Estimates of all elements of the above matrices are collected and assembled by BEA at the 484 sector level of detail. Before publication however, they are aggregated to about 360 sectors. BEA personnel responsible for this compilation were interviewed; their subjective estimates of uncertainty on all base year transactions are given in Appendix A.

Before proceeding with the Monte Carlo simulation, these data were aggregated to 90 and then to 30 sectors.* The 30 sector data base was used for the development and verification of all the computer programs. The 90 sector data base was used for the main simulation. This degree of aggregation was chosen for economic reasons (matrix inversion is an expensive N^3 operation) and because it corresponds most closely to the most widely distributed and used version of the BEA input-output tables. These are published at the 83 sector level of detail, while the 90 sector model used here retains more detail in the transportation and energy sectors of the economy.

Aggregating an input-output data base is a nontrivial operation since it must be done prior to the operations in (2.1-3). After aggregating the

* Names of sectors at each level of aggregation are given in Appendix B.

three matrices independently and summing to obtain T, X and A are computed using eqs. (2.1-1) and (2.1-2).

A 101 sector data base was also constructed by replacing the 5 energy sectors in the 90 order model by 16 energy supply and service sectors.* The rationale and development of the 101 sector model are described by Bullard and Sebald (1975). Its purpose is to more accurately predict energy consumption in future years by explicitly modeling fuel substitutability. In effect, this model recognizes that end uses of energy (space heating, lighting, air conditioning, etc.) are less substitutable than the fuels themselves by permitting the non-energy sectors to purchase only end uses of energy, while fuels are sold only to the end use sectors. Note that the former are very stable over time while the latter are not. The most variable coefficients or parameters involved in energy consumption are thereby confined to the few representing sales of fuels to the end use sectors, which can be estimated independently using models designed expressly for that purpose.

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* Names of sectors at each level of aggregation are given in Appendix B.

3.0 METHODOLOGY

3.1 Point of View for Stochastic Error Analysis

There are several ways to interpret this problem, and the point of view affects both the methodology and the interpretation of results. One way is to act as a simulator of BEA's activities from data collection through matrix inversion. In an alternative viewpoint, the analyst attempts an a priori determination of the effect of mathematical transformations on uncertain observations. In either case, this information enables the analyst to assess the usefulness of the data for modeling purposes. We have adopted the latter point of view.

Within this framework, the analyst receives signals from the economic system associated with each interindustry transaction as well as total output and value added. Actually, each of these signals from an industry is the sum of many signals from individual establishments. The signals appear to be independent; that is to say, the signals tell us little about their correlation.* The analyst's only information on these correlations comes from accounting identities requiring income to equal outgo.

Each signal is characterized by BEA in terms of upper and lower bounds and a "published value" representing their estimates of where the true value is mos likely to lie. We then characterize BEA's knowledge of the transactions as ran dom variables. The distributions are inputs for the Monte Carlo analysis which transforms them into a set of numbers comprising the solution set. Each elemen of the solution set is characterized by a set of statistics** which are then compared with the deterministic result.

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* Due to the size and complexity of the economic system, frequent measurement is economically prohibitive so no information is available from time series analysis.

** Mean, variance

Each input variable is first sampled independently, but some effort is then made to assure that the external balance conditions are satisfied. This is what BEA does in their deterministic approach, and we make a similar attempt with our Monte Carlo approach. It is unrealistic however, to completely simulate BEA's activities, many of which are judgemental, undocumented, and not reproducible. The specific shortcuts taken are detailed in section 3.4.

3.2 Sampling Random Variables

All of the basic data (transactions, industry output, final demands) are characterized as random variables having either normal or lognormal distributions.* As discussed in Appendix A, Section 4, entries which have been truncated to zero by BEA are modeled with a "folded normal" random variable, which is simply the absolute value of a normal random variable with mean 0. Non-zero cells are modeled using either normal or lognormal random variables with the former used in those cases where the published value is relatively accurate. In situations where the data is less well known, an analyst will tend to use a multiplicative factor to bound his estimate rather than an additive error bound. A lognormal distribution is appropriate in such a case because of its property of multiplicative symmetry about the median. That is, if X_0 is the median of a lognormal random variable X, then Prob. $(X \geq X_0 D)$ = Prob. $(X \leq X_0/D)$ for any factor D. For

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* In a few cases a negative entry in the data is modeled by the negative of a lognormal random variable (which necessarily takes only positive values). This set of circumstances is handled so much like the usual lognormal case that it is not discussed separately in what follows.

example, if an analyst states that his estimate has probability α of being correct within a factor of D, then a lognormal random variable with α = Prob. $(X_0/D \leq X \leq X_0D)$ will be used to model the situation.

This section outlines a procedure for sampling from random variables such that

1) The sample will be drawn from a folded normal, normal or lognormal population.

2) The distributions will be truncated to prevent samples that are absurd (e.g., negative transactions). Truncation eliminates samples in the upper and lower 0.15% tails in the normal and lognormal cases and in the upper 0.3% tail in the folded normal case. This corresponds to the percentage of probability outside 3 standard deviations from the mean in a normal population.

3) The expected value of the sampled result is equal to the published value, M, of the entry in question (except in the folded normal case where the published value is zero).

4) Before truncation, the random variable X from which we sample has a confidence interval defined by a parameter b, δ or D.

   a. Folded Normal Case

      Prob $(X \leq b) = .997$
      (i.e., b amounts to 3 standard deviations of the underlying normal random variable.

   b. Normal Case

      Prob $(\mu_X - \delta\mu_X \leq X \leq \mu_X + \delta\mu_X) = .997$
      (i.e., δ amounts to 3 standard deviations of X expressed as a fraction of the mean, $\mu_X = M$)

   c. Lognormal Case

      Prob $(X_0/D \leq X \leq X_0D) = .997$

In all three cases the sampling procedure is based on a standard normal random variable* (i.e., mean = 0 and variance = 1, denoted N(0,1)).

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* The standard normal random number generator used was the International Mathematical Statistical Library routine GGNRF. Tests of randomness and normality were performed for verification purposes and are described in Appendix D.

Truncation is achieved by sampling until a value r is obtained which is less than 3 in absolute value. In the folded normal case we set $\mu = 0$ and $\sigma = b/3$ so that $\mu + \sigma r$ is a sample from a truncated $N(\mu, \sigma^2)$ variable; the absolute value then satisfies the conditions for the folded normal sample. In the normal case we set $\mu = M$, $\sigma = \frac{\delta M}{3}$ and then $\mu + r\sigma$ is used as the normal sample.

The situation for X lognormal is slightly more complicated. In this case, $X = e^Y$ where Y is a normal random variable. Let μ and σ be the mean and standard deviation of Y.

Then the median X_0 of X is equal to e^{μ} so $\mu = \ln X_0$. Therefore,

$$\text{Prob } (X_0/D \leq X \leq X_0 D) = .997$$

implies that

$$\text{Prob } (\ln X_0 - \ln D \leq Y \leq \ln X_0 + \ln D) =$$

$$\text{Prob } (-\ln D \leq Y - \mu \leq \ln D) = .997$$

so that

$$\ln D = 3\sigma.$$

Since we want the mean value to equal the published value,

$\mu_x = e^{\mu + \frac{\sigma^2}{2}} = M$, we must set $\mu = \ln M - \frac{\sigma^2}{2} = \ln M - \frac{\ln^2 D}{18}$

To summarize, we sample for X in the lognormal case by obtaining a truncated standard normal random number, r, and setting $X = e^{\sigma r + \mu}$ where $\sigma = \ln D$ and $\mu = \ln M - \frac{\ln^2 D}{18}$. Comparing the three cases we have;

X Folded Normal	X Normal	X Lognormal
$\mu = 0$ $\sigma = b/3$	$\mu = M$ $\sigma = \delta M/3$	$\mu = \ln M - \frac{\ln^2 D}{18}$ $\sigma = \ln D/3$
X = truncated $ABS(N(\mu,\sigma^2))$	X = truncated $N(\mu,\sigma^2)$	X = truncated $e^{N(\mu,\sigma^2)}$

In the lognormal case the mean is not coincident with the median. To evaluate the error resulting from assuming they are equal, suppose an analyst gives a confidence interval for the true value T, in terms of his estimate M and a factor D. That is,

$$\text{Prob }(M/D \leq T \leq MD) = .997$$

where .997 is just the probability spanned by three standard deviations about the mean in a normal distribution.

We have modeled this situation with a random variable X with

$\mu_X = M$ and

$$\text{Prob }(X_0/D \leq X < DX_0) = .997$$

We want to show that

$$\text{Prob }(M/D \leq X \leq DM) \text{ is close to } .997.$$

In fact,

$$\text{Prob }(M/D \leq X \leq DM) =$$

$$\text{Prob }(\ln M - \ln D \leq Y \leq \ln D + \ln M) =$$

$$\text{Prob }(\mu + \frac{\sigma^2}{2} - 3\sigma \leq Y \leq 3\sigma + \mu + \frac{\sigma^2}{2}) =$$

$$\text{Prob }(\frac{\sigma}{2} - 3 \leq \frac{Y-\mu}{\sigma} \leq 3 + \frac{\sigma}{2})$$

Since $\frac{Y-\mu}{\sigma}$ is standard normal, we can find this probability in standard normal tables if we know σ. For a typical value of D such as

$$D = 8, \quad \frac{\sigma}{2} = \frac{\ln D}{6} = .35$$

Therefore,

$$\text{Prob }(M/D \leq X \leq DM) = \text{Prob }(.35 - 3 \leq \frac{Y-\mu}{\sigma} \leq 3 + .35) = .995\ ,$$

indicating that the error resulting from the assumption is negligible.

3.3 Aggregating Random Variables

Based on subjective uncertainty estimates made by BEA Personnel, probability distributions were defined at the 360 sector level of detail. Since simulations were done at the 101, 90 and 30 sector levels, aggregation was necessary. The means of the aggregated variables are easily obtained but specification of the distributions of the aggregate variables is a non-trivial task which was undertaken in the following way. Since all transactions, margins, etc. at the 368 order are in fact aggregates of data obtained initially from individual establishments grouped by 5 or 6 digit Standard Industrial Classification codes, the specification of a distribution for these aggregates was a crude assumption in itself. The basis for specifying the distribution at the 90 sector level is equally subjective, so we adopt the following convention. Assume that the variance, V, of each aggregated element is the sum of the variances of all its constituents. If $3\sqrt{V}$ is less than 40% of the aggregated mean, μ, assign a normal $N(\mu,V)$ distribution to the variable. If $3\sqrt{V}$ is greater than 40% of μ, a lognormal distribution is assumed. If μ equals zero, a folded normal distribution is used. This rule is simply a formalized reproducible characterization of a subjective assessment of input data uncertainty. It is felt that the subjective nature of the disaggregated uncertainty estimates did not warrant a more rigorous approach. For purposes of reproducibility, however, the adopted algorithm is detailed below.

The first step in aggregating is to compute the variances of the entries being aggregated:

Case 1: Folded Normal.* $V = (b/3)^2 (1- \frac{2}{\pi})$

Case 2: Normal. $V = (M\, \delta/3)^2$

Case 3: Lognormal.* $V = M^2 \left[\exp(\frac{\ln^2 D}{9}) - 1\right]$

As indicated above, the variances of the entries being aggregated are summed to obtain the variance of the aggregate entry. The decision to make the aggregate entry folded normal, normal or lognormal depends on the aggregate variance V and the aggregate published value M.

Case 1: $M = 0$. Here we assume that the constituent entries were also published zeros. The parameter b is chosen so that the variance of the resulting folded normal will equal the given aggregate variance V. This value is $b = \sqrt{V/(1 - 2/\pi)}$

Case 2: $M \neq 0$ and $3\sqrt{V}/|M| \leq .4$. These entries are modeled as normal with δ chosen so that the resulting variance will equal V. This value for δ is $\delta = 3\sqrt{V}/|M|$

Case 3: $M \neq 0$ and $3\sqrt{V}/|M| > .4$. Here we use a lognormal random variable specified by the parameter D chosen to be consistent with V. $D = \exp(3\sqrt{\ln(1 + V/M^2)})$*

3.3 Constructing the Transactions Matrix

Fig. 3-1 shows graphically the relationship between the matrices of transactions, (T), final demand (FD), imports (M) and gross domestic outputs (GDO).

$$\sum_{j=1}^{N} T_{ij} + \sum_{k=1}^{10} FD_{ik} - M_i = GDO_i \qquad (3.3\text{-}1)$$

These random variables are sampled from normal or lognormal distributions as described above. Each element in the first row (i=1) is sampled first independently, just as BEA analysts receive these values from apparently independent sources. Since eq. (3.3-1) is an external balance condition that

*See appendix C for details.

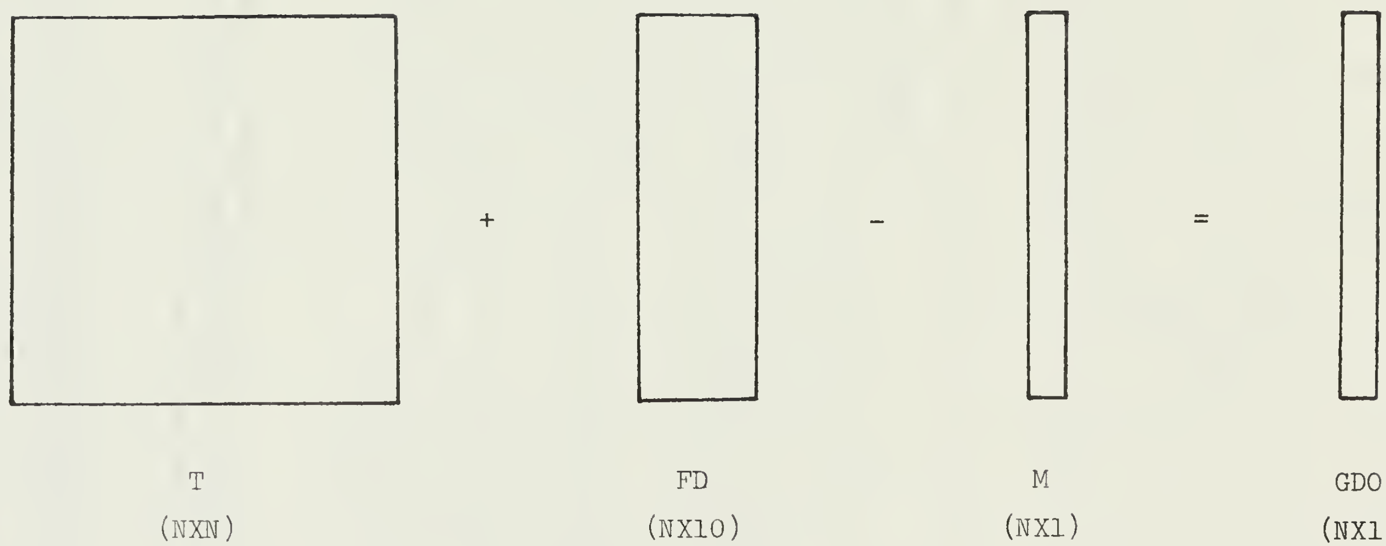

Fig. 3-1 Base Year Transaction Data

is not satisfied in general, we force this condition to be satisfied in much the same manner as BEA does. The lognormally distributed variables in the row are generally those obtained from unreliable sources or computed using surrogate variables. Therefore these values are scaled proportionately to satisfy eq. (3.3-1).*

Proceeding in this manner through N rows, a complete data set is constructed satisfying row constraints. The rows are not independent, however, because the value of all outputs (GDO) of a sector must equal the value of all commodity inputs (from the other N sectors) plus "value added" (a term, VA, accounting for wages, taxes, and profit). VA is measured independently by federal agencies and provides BEA analysts with another external condition to satisfy. Their method for satisfying this was too complex to model, so a simpler check had to be devised for this Monte Carlo study.**

The method employed is based on the response of the BEA's director of the I-O study to the following question: "If the criterion for terminating the iterative process of balancing the I-O table were based on uncertainty of the VA values, how much could be tolerated?" The answer indicated that out of 90 sectors, at least 88 must be within ±20% of the "true" value.*** If the condition was not met, the matrix was rejected. This condition was never violated in the actual simulation.

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* In fact, BEA analysts actually estimate many of these uncertain values by computing the difference between GDO and the sum of the well known (normally distributed) variables and allocating proportional to some surrogate variables (e.g. employment).

** In the 1967 input-output study, consistency between row and column sums was assured by assigning responsibility for individual sectors to different analysts and after each independently estimated initial row values, the resulting columns were presented to each analyst for independent verification. After many iterations and some undocumented judgement decisions, the "published" values were agreed upon.

*** Philip M. Ritz (1976) Interindustry Economics Branch, Bureau of Economic Analysis, U. S. Department of Commerce, personal communication.

Next, the terms in eq. (3.3-1) are used to compute the coefficients

$$A_{ij} = \frac{T_{ij}}{GDO_j}$$

and the Leontief inverse matrix $(I-A)^{-1}$ is finally calculated. Aside from checking the eigenvalues of A, there is no a priori check that can be performed to guarantee positivity of the inverse matrix.* Therefore, each inverse matrix is checked after it is computed to verify that every element is greater than zero. If it fails the test, all the randomly selected variables T, FD, M, GDO are discarded and a new set is selected. This is exactly the procedure employed by BEA. Again, the simulation was completed without this condition being violated.

3.4 Results Saved for Analysis

The simulation described here is expensive from a computational point of view since matrix inversion is an N^3 operation. For this reason, every simulated Leontief inverse matrix was saved on tape so it would be available for future analysis if necessary.**

For purposes of this analysis, our attention was focused on the means, variances and confidence intervals for the elements of $(I-A)^{-1}$ and selected subsets and linear combinations thereof. To calculate these, it was necessary to save a set of sufficient statistics on disk after each iteration, the running sum and the sum of the squares for each element of the following set of results which we shall denote by Ω:

---

* If all variables were expressed in current-year dollars, some a priori tests are available. In the general case such as this one, where the energy sector outputs are expressed in physical units, no such tests exist.

** The tape will be delivered to EPRI under separate cover.

1. The entire $(I-A)^{-1}$ matrix;

2. The total primary energy intensity vector, ε; and

3. The sector output vector, X.

The total primary energy intensity vector is a linear combination of the energy rows of $(I-A)^{-1}$, and a typical element ε_j represents the amount of basic energy resources required directly and indirectly to produce one unit of output from sector j for final consumption.* The sector outputs X are computed from the simulated $(I-A)^{-1}$ matrix using the base year domestic final demands as weighting factors:

$$X_i = \sum_j (I-A)^{-1}_{ij} \left(\sum_{k=1}^{10} FD_{kj} - M_j \right) \qquad (3.4\text{-}1)$$

This is done because I-O models are frequently employed to estimate total secto outputs corresponding to a specified final bill of goods, and a significant amount of additional error cancellation may be achieved.

In order to ascertain the nature of the distribution of typical random variables, each simulated value was saved for source results. The variables saved were X, ε, and the electricity sector row of $(I-A)^{-1}$. Goodness of fit tests performed on these variables are described in Section 4.

Finally, since most applications of the particular models examined are in the area of energy policy analysis, it was decided to save sufficient stat-

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* The energy rows utilized are those corresponding to coal, crude oil and gas, and the fossil fuel equivalent of hydro and nuclear electricity: $\varepsilon_j = (I-A)$ $+ (I-A)^{-1}_{2j} + 0.6\ (I-A)^{-1}_{4j}$

istics for recovering covariances of the energy sector rows of $(I-A)^{-1}$. Since all possible linear combinations were not of interest - only row and column combinations - storage requirements were considerably reduced. It was sufficient to save the running sums of products of all pairs of entries appearing together in such linear combinations. If other combinations are ever needed, they will be recoverable from $(I-A)^{-1}$ matrices saved on an archive tape as described earlier.

With this set of results it is possible to estimate the total energy requirements to meet arbitrarily specified final demands, and to compute linear combinations of energy intensities similar to the "total primary" one described earlier.

3.5 Stopping Rule

One of the major difficulties associated with Monte Carlo simulation is knowing how many runs will be required to attain reasonable confidence intervals on the results of the simulation. There are two major problem areas. If one is considering whether or not to use Monte Carlo techniques, an estimate of the required number of runs is crucial to determination of simulation costs. It may be, for example, that reasonable confidence intervals may require a prohibitively expensive number of runs. The second problem arises after the decision has been made to use Monte Carlo methods. One needs to know when enough runs have been made.

In the first problem area, present practice dictates running several small scale simulations of a similar nature to the one of interest in order to be able to extrapolate the number of runs in the smaller cases to the probable runs needed in the larger. In the second area, good statistical practice dictates that before taking any samples, one must determine how to stop sampling in a way that doesn't bias results. Executing additional runs if the resulting confidence intervals are too large is considered unwise since one runs the risk of biasing the simulation results by stopping when the desired outcome occurs.

In this section we present a method for determining, based on a very small number of runs, the proper number of total runs the simulation should require. The method properly elucidates the cost/benefit tradeoff between the cost of additional runs and the benefits of increased accuracy. Information is displayed to the analyst in a way that facilitates his making a judgement on the proper number of runs to be made. Since this method is based on just the first few runs, biasing of the simulation is not a problem. Based on a very small number of runs, it is also a cost effective way to decide whether a Monte Carlo analysis is economically feasible.

In section 3.5.1 we outline the approach used, in section 3.5.2 a brief sketch of the mathematics involved is given, followed by an example in section 3.5.3. Mathematical derivations are given in section 3.5.4.

3.5.1 An Outline of the Approach

Suppose a relatively small number of simulation runs have been made and unbiased estimates of the second order statistics of all elements of a set of results, Ω, have been calculated. Since the estimates are themselves random quantities, one can determine an interval about each estimate which contains the population value (e.g. mean or variance) with a certain probabilit
These intervals are called confidence intervals (CI) and we shall interest

ourselves in intervals for which the corresponding probability is .95. Fig. 3.5-1 describes the situation within which one must interpret the results of a simulation. Generally, then, the simulation output is given in two ways: the unbiased estimates are tabulated and their confidence intervals (e.g. 95%) are also given. Our strategy will be to make a few runs and then based on the resulting estimates and CI's, determine how many runs the entire simulation will require. Two major effects occur with increasing sample size. First, the estimates $\hat{\mu}$ and $\hat{\sigma}$ will move around, ultimately converging to the correct values. Second, the width of the CI's will decrease monotonically to zero as the number of runs goes to infinity. For purposes of the stopping rule, we have chosen to quantify the resulting simulation accuracy by monitoring a histogram of*

$$B^{+} \triangleq \frac{3\hat{\sigma} + U}{\hat{\mu}}$$

for each stopping point considered. The algorithm therefore:

1) Draws a histogram of the actual B^{+} values after an initial number of runs, m_1.

2) On the basis of the information after m_1 runs, draws a histogram of predicted B^{+} values after m_2 runs, where $m_2 > m_1$ denotes a possible stopping point for the full blown simulation.

3) Step 2 for various m_2.

Typical results are displayed in fig. 3.5-2.

3.5.2 A Mathematical Overview

As before, we denote the matrix of simulation output variables by Ω. Each $X \in \Omega$ has an unknown distribution which is at best only approximately normal.

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*The symbols are defined in Figure 3.5-1.

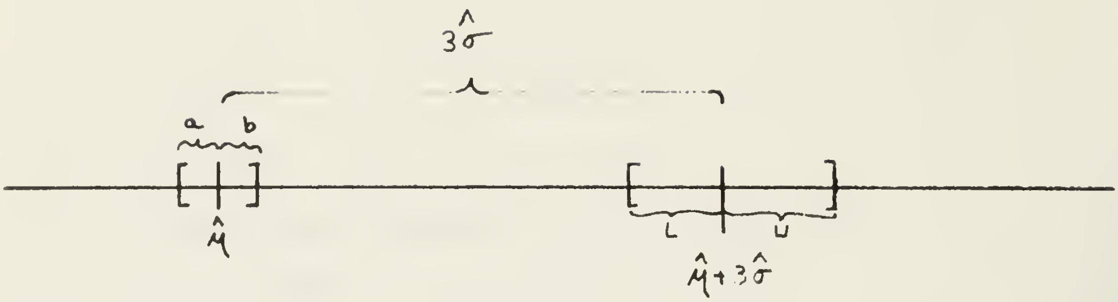

Figure 3.5-1 Mean and Variance Estimates and their Confidence Intervals

$\hat{\mu}$ = unbiased estimate of the mean of the underlying distribution a and b are the upper and lower confidence interval lengths for $\hat{\mu}$.

$\hat{\sigma}$ = unbiased estimate of the standard deviation of the underlying distribution

U and L are the upper and lower confidence interval lengths for $3\hat{\sigma}$.

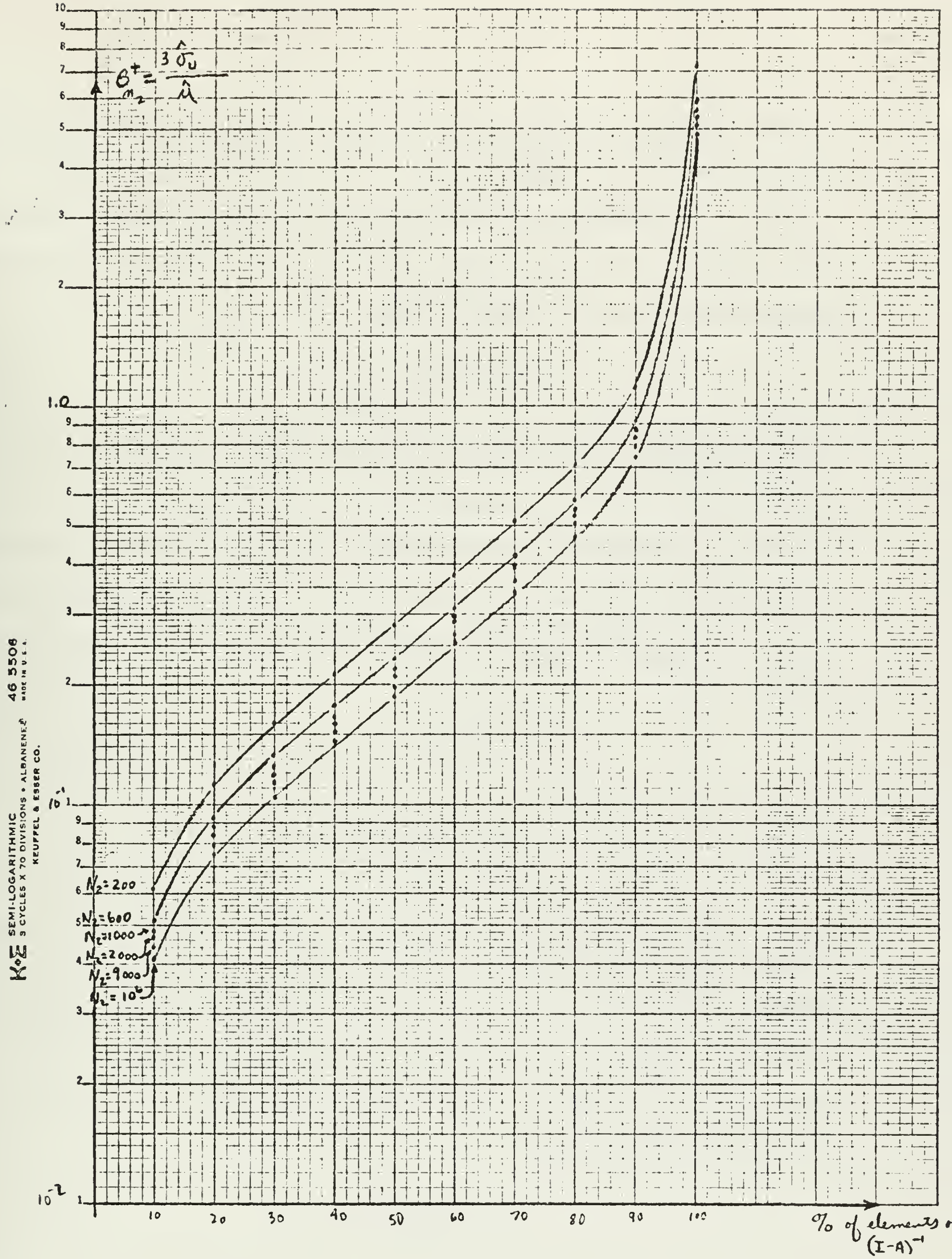

Figure 3.5-2 Stopping Rule Output Histograms for the 90 Order Simulation.

We denote the set of samples of each $X \varepsilon \Omega$ by $\{x_i\}$. Although what follows could be done for the unknown density of $X \varepsilon \Omega$, this approach involves inaccuracies in the determination of the fourth central moment and is computational quite expensive. Instead we have chosen to convert each $X \varepsilon \Omega$ to a normal random variable Z by the transformation

$$z_i \triangleq \frac{1}{10} \sum_{j=10(i-1)}^{10i} x_j \tag{3.5-1}$$

Since the samples x_j are independent and identically distributed (iid) the central limit theorem implies that the z_i are approximately $N(\mu_x, \sigma_x^2/10)$. All statistical evaluations will be performed on Z and the results will be back-transformed via (3.5-1) to X. Since the Z's are normal, the unbiased estimates for their mean and variance are given by the well known relations [Winkler & Hayes (1970)]

$$\hat{\mu} = \frac{1}{n} \sum_{i=1}^{n} z_i \tag{3.5-2}$$

$$\hat{\sigma}^2 = \frac{\sum_{i=1}^{n} (z_i - \hat{\mu})^2}{(n-1)} \tag{3.5-3}$$

where $n = m/10$ is the number of Z sample points and m is the total number of ru We assume that even after the initial m_1 runs, the CI around $\hat{\mu}$ is very small. This has empirically been verified as a valid assumption and it permits us to evaluate B^+ for the Z variables by only worrying about the upper CI on σ which we denote by $\hat{\sigma}_u^2$. Again due to the normality of the Z's, $\hat{\sigma}_u^2$ is well known to be [Winkler & Hayes (1970)]

$$\hat{\sigma}_u^2 = \frac{(n-1)\, \hat{\sigma}^2}{\chi(.975;\, n-1)}$$

where $\chi_{(.975_j\ n-1)}$ denotes the value in a chi-square distribution with n-1 degrees of freedom cutting off the upper .975 of sample values.

B^+ for the Z variables is then given by

$$B^+_n = \frac{3\hat{\sigma}_{Un}}{\hat{\mu}_n} = \frac{3\hat{\sigma}_n}{\hat{\mu}_n}\sqrt{\frac{(n-1)}{\chi^2_{(.975_i\ n-1)}}} \qquad (3.5\text{-}5)$$

where the subscript n denotes the number of Z sample points in the simulation. To evaluate $B^+_{n_2}$ we shall require $\hat{\mu}_{n_2}$ and $\hat{\sigma}_{n_2}$.* Even with relatively small n, $\mu_{n_2} \approx \mu_{n_1}$ since the CI even at n_1 is very short. Such is not the case with $\hat{\sigma}_{n_2}$. We can, however, upper bound $\hat{\sigma}_{n_2}$ if $\hat{\sigma}_{n_1}$ is known by noting that** $\frac{(n_2-n_1-1)n_1}{(n_2-n_1)(n_1-1)}$ has an F distribution with $n_2 - n_1$ and n_1 degrees of freedom where $Q \triangleq \frac{\hat{\sigma}^2_{n_2}}{\hat{\sigma}^2_{n_1}}$. We can then determine K such that

$$P\{Q \leq K\} = .50 \qquad (3.5\text{-}6)$$

and then use

$$\hat{\sigma}_{n_2} = \sqrt{K}\ \hat{\sigma}_{n_1} \qquad (3.5\text{-}7)$$

in (3.5-5). It is clear from (3.5-5) and (3.5-7) that for n_1 and n_2 fixed, the $B^+_{n_2}$ is linearly related to $B^+_{n_1}$ for each Z. The histogram of Figure 3.5-2 can then be generated by evaluating the actual histogram for B^+ after

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* $N_i = M_i/10$

** This is proved in section 3.5.4.

n_1 runs, fixing n_2, evaluating the constant multiplier, C, and multiplying each point in the histogram by C. In particular,

$B^+_{n_2} = C \ B^+_{n_1}$ Where

$$C = \sqrt{K\left(\frac{n_2 - 1}{\chi^2(.975;\ n_2-1)}\right)\left(\frac{n_1 - 1}{\chi^2(.975;\ n_1-1)}\right)^{-1}}$$

and K is given by (3.5-6)

The choice of the probability .5 in (3.5-6) is not arbitrary. We are interested in predicting the histogram of $B^+_{n_2}$ from the histogram $B^+_{n_1}$. For each $Z \varepsilon \Omega$, define an indicator function I_Z as follows:

$$I_Z = \begin{cases} 1 \text{ if } Q_Z > K \\ 0 \text{ otherwise} \end{cases}$$

The numbers of Z's for which $Q_Z > K$ is then equal to

$$N \triangleq \sum_{Z\varepsilon\Omega} I_Z$$

Provided .50 is used in (3.5-6), the expected value of $N = \sum_{Z\varepsilon\Omega} E\{I_Z\} = \frac{1}{2}|\Omega|$ where $|\Omega|$ denotes the number of elements in the set Ω. The histogram of $B^+_{n_2}$ can be predicted from that for $B^+_{n_1}$ if the behavior around the decile points can be quantified. The fact that $E\{N\} = \frac{1}{2}|\Omega|$ indicates that the set of points around each decile in the histogram of $B^+_{n_1}$ should behave in the following way. The deciles of $B^+_{n_2}$ are approximately C times the deciles of $B^+_{n_1}$ since for large $|\Omega|$ roughly half of the points around each $B^+_{n_1}$ decile is expected to change by more than a factor of C with the completion of n_2 runs. The other half is expected to change by less than a factor of C. Provided some degree of independence exists among the entries, the decile at n_2 should therefore be approximately c times the decile at n_1. The final step simply uses (3.5-1) to convert the $B^+_{n_2}$ histogram of values for the Z variables in Ω back to a histogram for the associated X variables. This just amounts to multiplying the decile values of B^+ by $\sqrt{10}$ since $\sqrt{10}\sigma_z = \sigma_x$ and $\mu_z = \mu_x$.

3.5.3 An Example

As an example, we shall discuss the actual 90 order stopping rule results. After 200 runs, the histograms of Figure 3.5-2 were generated.* As discussed above, $\beta^+ = 3\ \hat{\sigma}_U / \hat{\mu}$ was chosen as the ordinate since it is a useful measure of the variability of each element in the result set. This figure predicts the variability of results to be expected after different possible stopping points. The diminishing returns for increasing the number of runs is evident from a comparison of the marginal benefit by increasing from 200 to 600 runs to that obtained by increasing from 1000 to 2000 runs. This format permits the analyst to decide at which point the marginal benefit no longer justifies the increased cost. Clearly, this requires a non trivial judgement by the analyst. Based on the relatively high cost per iteration in this simulation, 1000 was chosen as the stopping point.

In an attempt to quantify the accuracy of this stopping rule, comparisons of predicted and actual histograms were made at the 90 order. Results are given in Table 3.5-1. Similar tests were done at the 30 order where actuals were compared with predictions based on only 100 runs, and very small errors were observed.

*Although similar histograms for the total primary vector, ε, and GDO were used in the determination of the proper stopping point, for purposes of this example, we shall concentrate on Figure 3.5-1.

200	400		600		800		1000	
ACTUAL	PREDICTED	ACTUAL	PREDICTED	ACTUAL	PREDICTED	ACTUAL	PREDICTED	ACTUAL
0.6149E-01	0.5405E-01	0.5562E-01	0.5134E-01	0.5364E-01	0.4987E-01	0.5230E-01	0.4888E-01	0.5056E-01
0.1121E 00	0.9653E-01	0.1016E 00	0.9360E-01	0.9745E-01	0.9091E-01	0.9445E-01	0.8911E-01	0.9264E-01
0.1605E 00	0.1410E 00	0.1452E 00	0.1340E 00	0.1383E 00	0.1301E 00	0.1343E 00	0.1276E 00	0.1307E 00
0.2128E 00	0.1671E 00	0.1930E 00	0.1777E 00	0.1857E 00	0.1726E 00	0.1788E 00	0.1692E 00	0.1749E 00
0.2801E 00	0.2463E 00	0.2564E 00	0.2339E 00	0.2447E 00	0.2272E 00	0.2370E 00	0.2227E 00	0.2319E 00
0.3764E 00	0.3308E 00	0.3349E 00	0.3143E 00	0.3173E 00	0.3052E 00	0.3081E 00	0.2992E 00	0.3019E 00
0.5075E 00	0.4461E 00	0.4552E 00	0.4238E 00	0.4343E 00	0.4116E 00	0.4233E 00	0.4035E 00	0.4141E 00
0.7086E 00	0.6229E 00	0.6311E 00	0.5917E 00	0.6028E 00	0.5747E 00	0.5851E 00	0.5633E 00	0.5722E 00
0.1111E 01	0.9753E 00	0.9874E 00	0.9274E 00	0.9619E 00	0.9008E 00	0.9356E 00	0.8830E 00	0.9092E 00
0.7213E 01	0.6338E 01	0.5844E 01	0.6021E 01	0.5376E 01	0.5848E 01	0.5288E 01	0.5732E 01	0.4959E 01

TABLE 3.5-1 A Comparison of $(I-A)^{-1}$ Stopping Rule Predicted Histograms and Their Corresponding Actuals for the 90 Order Simulation

3.5.4 Mathematical Derivations

We first demonstrate in this appendix that $\frac{(n_2 - n_1 - 1)n_1}{(n_2 - n_1)(n_1 - 1)}$ Q has an $F_{(n_2 - n_1;\, n_1)}$ distribution where $Q \triangleq \frac{\hat{\sigma}^2_{n_2}}{\hat{\sigma}^2_{n_1}}$ and then outline the method for calculating the K of (3.5-6).

Consider the event $R \triangleq \left\{ \frac{\hat{\sigma}^2_{n_2}}{\hat{\sigma}^2_{n_1}} \leq K \right\}$

where $$\hat{\sigma}^2_n = \frac{\sum_{i=1}^{n} y_i^2}{n-1} \qquad \text{and } y_i = z_i - \hat{\mu}_{z_i}$$

$$\therefore \quad R = \left\{ \sum_{i=1}^{n_2} y_i^2 \leq (n_2-1)\, K\, \hat{\sigma}^2_{n_1} \right\}$$

Since $$\sum_{i=1}^{n_1} y_i^2 = (n_1-1)\, \hat{\sigma}^2_{n_1} \quad ,$$

$$R = \left\{ \sum_{i=n_1+1}^{n_2} y_i^2 \leq [(n_2-1)\, K - (n_1-1)]\, \hat{\sigma}^2_{n_1} \right\}$$

Dividing both sides of the inequality by $(n_2-n_1)\, \hat{\sigma}^2_{n_1} \left(\frac{n_1-1}{n_1}\right)$ we obtain:

$$R = \left\{ \frac{\left(\sum_{i=n_1+1}^{n_2} y_i^2\right)/(n_2-n_1)}{\left(\sum_{i=1}^{n_1} y_i^2\right)/n_1} \leq \left[\frac{(n_2-1)K - (n_1-1)}{n_2-n_1}\right] \left(\frac{n_1}{n_1-1}\right) \right\} \quad *$$

For large samples, $\hat{\mu}_z$ is much closer to the mean than any given sample point,

z_i, and therefore y_i are $N(0,\sigma^2)$ and iid. Introducing $q_i = \frac{1}{\sigma} y_i$ in * we obtain

$$R = \left\{ \frac{\left(\sum_{i=n_1+1}^{n_2} q_i^2\right)/(n_2-n_1)}{\left(\sum_{i=1}^{n_1} q_i^2\right)/n_1} \right. \leq \left[\frac{(n_2-1)K - (n_1-1)}{(n_2-n_1)} \right] \left(\frac{n_1}{n_1-1}\right) \qquad **$$

where $q_i \sim N(0,1)$. Since $q_i \sim N(0,1)$ and q_i are iid, both the numerator and denominator of the LHS of ** are χ^2 random variables divided by their respective degrees of freedom, and the LHS of * and ** have an $F_{(n_2-n_1;n_1)}$ distribution [Winkler and Hayes (1970)]. Therefore, $\frac{(n_2-n_1-1)n_1}{(n_2-n_1)(n_1-1)}$ Q has an $F_{(n_2-n_1;n_1)}$ distribution. QED

We shall now determine K such that P {Q<K} = .5. By **, this event is equivalent to

$$P\left\{ F_{(n_2-n_1;n_1)} \leq \frac{n_1}{n_1-1} \cdot \left[\frac{(n_2-1)K - (n_1-1)}{n_2-n_1} \right] \right\}$$

Therefore we simply find α such that $P\left\{ F_{(n_2-n_1;n_1)} \leq \alpha \right\} = .5$ and find K from

$$K = \left[\frac{\alpha(n_1-1)(n_2-n_1)}{n_1} + (n_1-1) \right] \frac{1}{n_2-1}$$

4.0 ANALYSIS OF RESULTS

The basic results of the simulation will be given for the 90 order I-O matrix. This includes information on bias relative to published values, variance measures and their relation to error bounds, the sensitivity of the results to uncertainties on the variances of the underlying BEA data and the effects of aggregating to 30 sectors and disaggregating to 101 sectors. As a prelude, we begin by discussing the goodness of fit tests which were required to verify some distributional assumptions inherent in the simulation.

4.1 Goodness of Fit

The methodology for the goodness of fit tests was developed by Stephens (1976), who describes a test for normality based on the Cramer-von Mises statistic which may be employed when the population mean and standard deviation are not known. Stephens' test compares a given sample distribution function to a normal distribution with mean and standard deviation given by the sample mean and sample standard deviation. Included in Stephens' paper is a table of significance levels for the statistic given the hypothesis that the random variable being tested is normal. Thus, a test of normality may be made by calculating the value of Stephens' statistic for a given sample and comparing it to the tabulated values which characterize normal behavior.

The first series of tests using this method was made to test the normality of the Z random variables defined by averaging every ten consecutive sample points obtained for the entries in the simulation results. In all, 270 of these random variables were tested, one for each entry in the electric utility sector row of $(I-A)^{-1}$, the total primary energy vector, ε, and the total output vector, X. Table 4-1 shows the upper tail percentage points calculated by Stephens along with the observed percentages of the Z random variables which fell into the various categories.

OBSERVED PERCENTAGE	16.0	12.22	6.29	3.33	1.
NORMAL PERCENTAGE	15.0	10.0	5.0	2.5	1.
STEPHENS' STATISTIC	.091	.104	.126	.148	.

Table 4-1. A Comparison of Observed and Theoretical Upper Tail Percentage Points for Goodness of Fit Tests on the Random Variables Z.

For example, Stephens predicts that 10% of all normal samples will achieve sample statistics larger than .104; we observed 12.2% above that mark. Even if the 270 random variable being tested are interdependent, the expected value of the observed percentages should equal the theoretical percentages if the normality hypothesis is satisfied. Thus the results are very reassuring and seem to justify treating the average variables as normal.

A second series of tests was undertaken to examine the distributional properties of the raw data for the same 270 entries. In the absence of averaging there is little reason to suspect that these random variables are normal. However, the results were surprising in that very many of the 270 sample statistics were small and therefore indicate good fit to a normal distribution curve. Those entries that displayed decidedly non-normal behavior were virtually all unimodal but slightly skewed to the right. It is interesting to conjecture why some entries seem to be roughly normal while others are not; perhaps in the process of inversion some elements of $(I-A)^{-1}$ get a better mix of elements of the A matrix. At any rate it is useful to know that the entries are all more or less unimodal and symmetric. If such is the case then 3σ may be conveniently employed as an error bound on the distance from the mean, μ. While Chebychev's inequality guarantees that

$\mu \pm 3\sigma$ contains at least 89% of the total probability in an arbitrary distribution, this percentage rises to 99.7 in the normal case. Presumably the percentage is also high for any random variable whose density function is roughly unimodal and symmetric. For all but one of the entries examined here, at least 99% of the sample points fell within three sample standard deviations of the sample mean. Thus, $3\hat{\sigma}$ may be thought of as an approximate bound on deviation from the mean for the entries in the simulation results, even if many of those entries are not very close to being normal.

4.2 Confidence Intervals

This section discusses the precision of the sample statistics obtained for various simulation results in light of the goodness of fit tests just discussed. Because the Z variables are approximately normal, standard techniques may be used to derive confidence intervals for the mean and variance of a Z variable and hence for the mean and standard deviation of the associated entry. After 1000 inversions, a 97.5% upper confidence bound σ_u on the standard deviation σ of an entry is given by $\sigma_u = \hat{\sigma} * 1.16$. Thus, $\hat{\sigma}$ is a fairly good estimate of σ for any given entry.

The confidence intervals on the sample means are even smaller. In more than 90% of the entries in the inverse the population mean is within 2% of the sample mean with 95% confidence. All the entries of ε and X are accurate to **within** 1% with 95% confidence.

4.3 Variability of the Elements in the Result Set

Histograms of $3\hat{\sigma}/\hat{\mu}$ were prepared in order to show the relative amount of variability in the entries of the results set. Three such histograms, one for the whole inverse, one for ε and one for X, are displayed in Figure 4-1. For half of the entries of the inverse, $3\hat{\sigma}/\hat{\mu}$ is less than 20% while virtually all the entries of ε and X have $3\hat{\sigma}/\hat{\mu}$ less than 20%. The above discussion of confidence intervals suggests that these histograms would not change substantially if the sample statistics were replaced by the population means and standard deviations. Since these entries are roughly unimodal and symmetric, the histograms may then be taken as a good measure of the variabili in the entries of the various subsets of the results. The large decrease in variability from the elements of the inverse to the elements of X suggests that significant error cancellation occurs as linear combinations of many I-O coefficients are computed.

In addition to those discussed above, histograms for $\dfrac{(3\sigma_u + \hat{\mu} - p)}{p}$ and $\dfrac{(3\sigma_u + p - \hat{\mu})}{p}$, where p = published value, were also computed in order to relate p to the upper and lower bounds on the uncertainty in an entry. Because $\hat{\mu}$ is generally very close to p and because σ_u is only slighter larger than $\hat{\sigma}$, these histograms are very similar to the histograms for $3\hat{\sigma}/\hat{\mu}$ except that the values are all slightly larger.

4.4 Bias on Elements of the Result Set

In standard statistical language, bias is usually defined as the difference between the mean of an estimator and the true value of the quantity to be estimated. We use the term in a fundamentally different way to denote the difference between the mean of the simulation output variables and their corresponding published values. The mean values of the assumed distributions of each element of the transactions matrix are equal to their respective published values. One important result of the simulation is to determine the bias introduced by normalization and inversion in passing from the transactions matrix to $(I-A)^{-1}$.

Fig. 4-2 details histograms of the ratio of sample mean to published value, $\hat{\mu}/p$, for three important and disjoint subsets of the result set, viz the vectors of total output X and total primary energy intensity ε and the entire inverse $(I-A)^{-1}$. Three aspects are noteworthy:

1) Nearly all $\hat{\mu}$ cluster within 2% of their published values.

2) Within this cluster, $\hat{\mu}$ tends to have a positive bias more often than a negative one.

3) Essentially none of the $\hat{\mu}$ fall below 98% of their respective published values while, especially in the inverse, a small number of $\hat{\mu}$ range well above the published value.

The reason for this positive bias is unclear. The best explanation may be that transactions reported by BEA as zero were assigned a small positive value in the simulation to account for the fact that no transaction is known to be exactly zero (see Appendix C). The large percentage excess over the published value may result for the same reason, since an inverse element may be affected (percentagewise) quite significantly if its corresponding direct coefficient A_{ij} changes from zero to some finite value.

4-5. Sensitivity of Simulation Results to Assumptions on Input Uncertainties

Since the variances assigned to input quantities such as the transactions matrix, FD and GDO are only estimates of the true variances, simulation results have meaning only if small changes in these assumed variances, do not cause very large changes in simulation outputs. This sensitivity to changes in input variances was investigated by repeating the simulation with the standard deviations of all normal quantities doubled and dispersion factors on lognormal inputs doubled. Three major effects were noted:

1) The ratio of CI to $\hat{\mu}$, where CI is the length of the 95% confidence interval for the mean, was doubled by the factor of two increase in standard deviation.

2) The ratio of $3\hat{\sigma}$ to $\hat{\mu}$ doubled on the average by doubling the input standard deviations.

3) Increasing the input variability made the biases slightly more negative. This is thought to be the result of increasing simulation sensitivity to the larger elements of the transactions matrix and decreasing relative sensitivity to the smaller elements discussed in section 4.4.

Since output uncertainties only doubled with a factor of two increase in input uncertainties, the simulation is probably very stable with regard to assumptions on input variances.

The absolute magnitudes of these results with doubled input uncertainties may be useful in assessing the general viability of I-O results applied far beyond the base year (the uncertainty of base year parameters increases over time). Moreover, if institutional factors make it unlikely (as some claim) that government can fairly estimate uncertainty of its own data, then these results show the effect of a 50% underestimate of the actual uncertainty.

4.6 The Effect of Aggregation

The effect of aggregating the 90 order model to 30 order was analyzed for two reasons:

1) It was felt that although variances at the 30 order were smaller than those of the 90 order due to the aggregation, more error cancellation should exist at the 90 order where more input elements combined to form elements of X, ε and $(I-A)^{-1}$.

2) Since much I-O work is done at the 360 order, it is of interest to determine whether the expansion of the simulation to 360 order would likely require more than the 1000 runs used in the 90 order case.

Aggregation produced effectively no change in the simulation output uncertainties:

1) The ratio of $3\hat{\sigma}$ to $\hat{\mu}$ remained virtually unchanged by the aggregation. This implies that the two effects mentioned above virtually cancel one another.

2) The already very small biases of Fig. 4-2 were made slightly more negative by aggregating to the 30 order.

3) Since the ratio $\sigma_u/\hat{\sigma}$ is a function only of the number of simulation runs, it is unaffected by aggregation.

These results give no indication that more than 1000 runs would be needed in the 360 sector case.

4.7 Results for the 101 Sector Model

As discussed in Section 2.2 above, the purpose of the 101 order model is to trade increased base year uncertainty for increased parametric stability over time. The purpose of the 101 order simulation was to measure the increase in base year uncertainty over the 90 order model. Comparison of 90 order and 101 order histograms for $\hat{\mu}$/published, $CI_{\mu}/\hat{\mu}$ and $3\hat{\sigma}/\hat{\mu}$ indicates virtually no change in $(I-A)^{-1}$, GDO and ε and a slight increase in $3\hat{\sigma}/\hat{\mu}$ for the energy related rows of $(I-A)^{-1}$. In particular, at the 90 order, 95.6% of the elements of the total primary energy intensities had $3\hat{\sigma}/\hat{\mu} < .15$ while at the 101 order, 94% were less than .15. This indicates a rather low cost in increased stability over time. This low cost is thought to be due to the fact that there are many more elements of the transactions matrix which are important to the energy embodied in a particular sector output*. Even though the uncertainty of each of these elements is greater in the 101 case, more of them combine so greater error cancellation occurs.

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* Consider natural gas to auto manufacturing. Natural gas is sold to perhaps eight energy products which in turn are sold to the automobile sector.

APPENDIX A. BASE YEAR UNCERTAINTY ESTIMATES

Clark W. Bullard

A.1 BEA DATA

A.1.1 INTRODUCTION

Input-output data form the basis for most structural analyses of the U.S. economic system. The massive tables of data provide a complete and internally consistent set of linear production functions for all sectors of the economy. But surprisingly, these analytical objectives and applications do not guide the efforts to acquire the data and compile the tables. Actually the input-output tables are constructed as a bridge between the national income and product accounts for selected base years in order to provide a "benchmark GNP" estimate for those years.*

It is important for the analyst using input-output (I-O) data for structural analyses to view the data from this perspective. Since it was not acquired primarily to support structural economic analyses, it places additional burdens on the analyst to verify the data's usefulness and relevance to his particular application.

Consider the most general type of application, where the analyst wants to predict sector outputs $\underline{X}$ needed to produce a final bill of goods $\underline{Y}$. To do this he premultiplies $\underline{Y}$ by the Leontief inverse matrix $(\underline{\underline{I}}-\underline{\underline{A}})^{-1}$ which is calculated from a matrix of direct coefficients $\underline{\underline{A}}$ for a (prior) base year input-output table.** The problem the analyst must address is: What is the uncertainty $\Delta\underline{X}$ on the result $\underline{X}$ given

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*Data for 368 sectors are published by the U.S. Department of Commerce (1974a)

**For a more detailed discussion of input-output analyses see Leontief (1941).

uncertainty $\Delta\underline{\underline{A}}$ in the input-output coefficients? Actually $\Delta\underline{\underline{A}}$ may result from 1) base-year measurement error and 2) changes in the actual $\underline{\underline{A}}$ since the base year. In this paper we are concerned only with the former.

Quantitative methods for treating this general error analysis problem are of three types. The first uses the condition number of $(\underline{\underline{I}}-\underline{\underline{A}})$ to obtain a bound on the norm of $(\underline{I}-\underline{A})^{-1}$, resulting in an extremely conservative upper bound on parametric error magnification. An expression for true maximum upper bound on $(I-A)^{-1}$ was derived by Sebald (1973), and the relative importance of certain parameters to specific applications was determined. Even the true upper bound, however, was quite conservative in that it did not account for the (likely) possibility of error cancellation.

This report is limited to presentation of uncertainty estimates on I-O data used in calculating the direct coefficients $\underline{\underline{A}}$.

A.1.1.1 Sources of Error

Uncertainty in the I-O coefficients is related directly to several source of error in estimating interindustry transactions for the base year. Due to the exhaustive nature of I-O data, it originates from a variety of sources ranging from census questionnaires to judgemental guesses. Morganstern (1950 has categorized the various sources of error in economic data and most of his observations are relevant here. The total uncertainty on a particular transaction "measurement" will include effects of incomplete census coverage, reporting errors due to misunderstandings or outright lying, sampling errors inherent in surveys of firms, transcription or key punching errors, the possibility that forms are lost, classification errors (matching firms and products to SIC codes), and last but certainly not least, the problem of separating companies from establishments in processing returns from surveys or censuses.

A.1.1.2 Effects of Scale

The scale of this problem is what makes it unique. Due to the size and complexity of the system being modeled (the U.S. economy) measurments can be taken only at infrequent intervals and at great expense. Moreover, it takes whole institutions to obtain the measurements (e.g. the U.S. Census Bureau) so the user of the data is generally not the one who acquired it. Thus the burden borne routinely by persons who play the roles of data-taker and analyst is now split amoung bureaucracies. Part of that burden--responsibility for estimating parametric uncertainty and its effects on analyses--is sometimes never borne because of the way the roles and responsibilities of the bureaucracies are defined.

The mission of the Census Bureau is to produce statistics; the Bureau of Economic Analysis (BEA) takes these and others and produces accounting tables supporting a benchmark GNP estimate. The analyst would like to take these statistics and interpret them as observations of a physical system whose structure he would like to model. The statistics are often published in terms of 5 or 10 significant figures, but none of the hundreds or thousands of persons involved in deriving a statistic are responsible for estimating and documenting its uncertainty.

A.1.2 METHOD

Recognizing that actual measurements of interindustry transactions and other variables are made in the presence of "noise" (error sources), and that frequent measurements are impractical, we must rely on subjective estimates of uncertainty. Such estimates are best obtained at the level of detail at which the measurements are taken, but here too a compromise must be made. A single transaction in an I-O table may be the sum of millions of individual measurements of physical quantities; this report is based on interviews with personnel at BEA, Census, and other agencies near the top of this statistical pyramid.

A.1.2.1 Quantities Estimated

Uncertainty estimates were obtained on the three basic constituents of the interindustry transactions matrix. These were direct allocations, margins on domestic transactions, and transfers.* Independently, estimates were obtained for final demands, gross domestic outputs, and imports and exports.

In the next section, uncertainty estimates will be given for each of these categories of data.

A.1.2.2 Degree of Detail

Within the scope of this study it was possible to consider data inputs to the I-O tables at the 484-sector level of detail in many cases; and at the 368-sector level for the remainder. At the more detailed level, a magnetic tape was available from BEA which included notes for various direct allocations indicating the source of the data and the magnitude of the

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*Precise definitions of these terms are given by the U.S. Department of Commerce (1974b).

figure obtained from that source. This tape was scanned for notes identifying entries from the Census Bureau or other sources deemed equally accurate.* If more than 75% of the entry in the 368-order Direct Allocations matrix was from one of these sources, it was assigned the same uncertainty as census data. Estimates of uncertainty for all other data were made at the 368-level of detail as described in the next section.

A.1.2.3 Interview Techniques

Many agency personnel seemed well-prepared and sometimes even anxious to assign quantitative estimates of uncertainty to the statistics for which they were responsible. Others were quite reluctant, citing the fact that the "correct" answer was not known and only one measurement had been taken so there was inadequate information on which to base an answer. While this latter group was probably more correct in their assessment of the situation, it should be remembered that such a statement could be used as an "excuse" for covering up error levels that might reflect badly on one's job performance. In virtually every case, those interviewed responded with a quantitative answer to a question of the form "If God appeared and told the correct number to the commander of a firing squad, and if that commander asked you to estimate error bounds for your published figure and threatened to kill you if the correct figure lay outside the bounds...What would you estimate?"

During the course of interviews with persons relying on the same data sources, and with persons responsible for producing that source data, I was able to arrive at what I believe to be an internally consistent set of uncertainty estimates. All results presented in this report may be attributed to the

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*These sources are <u>Minerals Yearbook</u>, <u>Census of Mineral Industries</u>, <u>Census of Manufactures Table 7A</u>, <u>Census of Transportation</u>, <u>Census of Business</u>, Interstate Commerce Commission, and Civil Aeronautics Board publications.

author, although footnotes are used to identify the persons whom I interviewed to obtain information and impressions. Given the nature of the strong institutional pressures for downward bias in these estimates,* I do not expect that the pressures for conservatism that I offered in phrasing my interview questions provided a significant counteracting force.

A.1.2.4. Bias

It is expected that uncertainty estimates obtained from such "top of the pyramid" interviews will be biased downward, since a BEA employee (say) will be reluctant to question the Census Bureau's estimate of the total U.S. steel production unless he has conflicting statistics from somewhere else. Since the Census Bureau has a virtual monopoly on such statistics, the latter situation is impossible; since the BEA employee has barely the resources to do his own job, he cannot begin to duplicate the efforts of the Census Bureau so the former situation never arises either. Simply stated, if one bureaucracy publishes a seven-significant-figure statistic that cost a million dollars to derive, the humble bureaucrat in another agency, with his own problems to worry about, is unlikely to seriously challenge the figure.

Possible treatments for this problem of bias will be discussed in the last section.

A.1.2.5 Effect of Numerical Magnitude

Development of I-O data involves much work within established, or relatively well-known, control totals. For this reason, and since the work is done primarily within an accounting framework, the largest numbers

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*See Morganstern (1950).

usually receive the most attention and are the best known, and the "residual" between the well-known components and the control total is often distributed among other categories using some kind of estimation algorithm. The only exception to this general "rule" occurs when the figure involved has a significant impact on the value of GNP; then, though small, the figure may become the subject of further analysis and refinement.

A.1.3. UNCERTAINTY ESTIMATES

In this section, estimates will first be presented at the 368-sector level of detail. This was the level of disaggregation at which most of the persons interviewed were most comfortable in assigning their subjective estimates of uncertainty.

As indicated earlier, all estimates of upper and lower bounds presented here may be attributed to the author. The discussion and footnotes indicate the source of my impressions and information.

Estimates of upper and lower bounds are given in two ways. The first is a fraction δ which denotes symmetric bounds around the published value of $\pm$ 100δ %. The second, applied in cases where the published value is less well known, is the factor D which when multiplied by the published value gives the upper bound, and whose inverse determines the lower bound. All bounds should be taken to represent a 99.7% confidence level.

A.1.3.1 Direct Allocations

"Good" Census-grade entries.* All transactions from one manufacturing sector to another are assigned $\delta = .05$, as are all other interindustry direct allocations obtained from Census Bureau sources. This figure is

*This information based primarily on interviews with Kenneth Hanson, Richard Chassey, Ruth Runyan, and Patrick Duck of the Census of Manufactures, Industry Division.

based on interviews with Census Bureau personnel who feel their techniques for circumventing problems associated with less than 100% coverage are well within these limits, and that internal cross-checks minimize reporting and related errors. The largest source of error here is suspected to be classification error; matching products and firms to SIC codes.

Agriculture sector rows.* Based largely on crop reporting surveys; estimate $\delta = .10$ except for certain transactions noted elsewhere (e.g. government final demand).

Agriculture sector columns.* Inputs from real estate, chemicals, and chemical fertilizer mining are known best from surveys and other sources; estimate $\delta = .10$. Directly allocated inputs from transportation and trade sectors were treated the same as margins, as described in sections 3,4. All other entries are based at least in part on farm expenditure surveys taken in 1955; assume $D = 2$ for all entries greater than 1% of gross domestic output for the sector. All smaller nonzero numbers scaled from $D = 2 \rightarrow D = 10$ as described in Sec. A.3.

Federal government purchases.** For both defense and non-defense purchases, the following assumptions apply: new construction inputs are based on a good data source, so assign $\delta = .05$; maintenance and repair construction is more subject to classification errors, so $\delta = .10$. All entries between $10 million and $50 million are assigned $\delta = .30$ unless otherwise specified below. Purchases less than or equal to $10 million are assigned $D = 2 \rightarrow 10$ as discussed in Sec. A.3.

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*Based primarily on interviews with Jerry Schluter, U.S. Department of Agriculture.

**Based primarily on interviews with Roy Seaton, Bureau of Economic Analysis.

Defense purchases are generally better known, due to more complete source data. Inputs from manufacturing sectors are assigned $\delta = .10$ if they exceed $50 million. Transportation inputs were derived from outdated formulae that applied poorly to the Southeast Asia situation in 1967 and are assigned $\delta = .50$. Other non-manufacturing inputs were assigned $\delta = .10$ if they were above the $50 million threshold.

Non-defense purchases of inputs from non-manufacturing sectors were less well known, and were assigned D = 3 if they exceeded one percent of total inputs and $D=3 \rightarrow 10$ if they were smaller. Manufacturing inputs below the $50 million threshold were treated the same. Transportation inputs were assigned $\delta = .30$.

State and local government purchases.* For health, welfare, education, and sanitation purchases, new construction and real estate inputs are assigned $\delta = .05$ since they are obtained from census sources. Together with wages, these inputs account for nearly 75% of all inputs. Other inputs are assigned $\delta = .25$ if they exceed 1% of total inputs, and $D = 1.5 \rightarrow 10$ as per Sec. A.3 if they are equal to or smaller than 1%.

For public safety purchases, new construction and real estate are assigned $\delta = .05$. Maintenance construction is known poorly; $D = 1.5$. Manufactured inputs greater than $2 million are assigned $D = 1.5$, and smaller inputs $D = 1.5 \rightarrow 10$. Non-manufactured inputs are assigned $D = 1.5$ for those greater than $10 million, and $D = 1.5 \rightarrow 10$ for the smaller ones.

Other state and local government purchases are also assigned $\delta = .05$ for new construction and real estate, but also $\delta = .05$ for maintenance construction since it is primarily highway maintenance which is a Census

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*Based primarily on interviews with John Wealty, Bureau of Economic Analysis.

number. Manufactured inputs greater than $5 million are assigned $D = 1.5$, and smaller figures $D = 1.5 \rightarrow 10$ as per Sec. A.3. Non-manufactured inputs greater than $50 million are assigned $D = 2$ and $D = 2 \rightarrow 10$ for smaller inputs.

Imports and exports.* Trade data for commodities (BEA sectors 1.00 - 64.00) are obtained from Census sources and are assigned $\delta = .05$. Transportation and wholesale and retail trade data, including margins, were assigned $\delta = .25$. Data on other items (services, etc.) involved in international trade were assigned $D = 2$, since they were obtained from balance of payments sample data. Small entries at the 368-sector level of detail, representing less than 1% of gross imports or exports were assigned $D = 2 \rightarrow 10$ as per Sec. A.3.

Inventory change. These figures are in general the least accurate of all final demand entries, and were assigned $\delta = .20$ for manufactured goods and $\delta = .40$ elsewhere.

"All other" direct allocations. Within the scope of this study it was impossible to identify those responsible for most entries in the input-output tables. Having taken care of most entries through interviews described above, the remainder were handled as a group. The algorithm was designed to assign very tight tolerances to any transaction comprising a high percentage of total outputs or inputs, and to any sector's output which "by definition" had to be assigned to a particular cell. For example, the algorithm had to assign a very tight tolerance to sales from new residential construction to gross private capital formation, so it would be compatible with the tolerance assigned to that sector's gross domestic output. There are numerous other instances where census data might identify

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*Based primarily on interviews with Robert Mangen, Bureau of Economic Analysis.

sales of butter to food processors or bakers, and the remainder is attributed to personal consumption expenditures. On the other hand, very small-magnitude transactions were assigned high uncertainty for the reasons discussed earlier.

The algorithm defined two fractions for each direct allocation: an input fraction, by normalizing with respect to the gross domestic output of the consuming sector; and an output fraction, by normalizing with respect to the gross domestic output of the producing sector. The algorithm proceeds with these tests in the following order, and assigning δ or D when the first condition is satisfied: if both fractions exceed .95 then $\delta = .01$, if only one exceeds .95 then $\delta = .02$; if both exceed .80, $\delta = .05$, if only one exceeds .80, $\delta = .10$; if either fraction exceeds .05, then $\delta = .20$; if either exceeds .01, then $D = 1.5$. If both are smaller than .01 it assigns $D = 2 \rightarrow 10$ as per Sec. A.3.

A.1.3.2 Gross Domestic Output*

These figures are the best known because they are from the Census or other equally reliable sources (e.g., IRS) and are assigned $\delta = .01$. The largest errors here probably stem from classification problems and possible confusion between company and establishment-based data.

A.1.3.3 Transfers.**

If both the row and column sectors were manufacturing sectors, the

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*Based primarily on interviews with Gene Roberts and Phil Ritz, Bureau of Economic Analysis, and with Kenneth Hanson, Census of Manufactures Industry Division.

**Based primarily on interviews with Kenneth Hanson, Census of Manufactures Industry Division.

source of this data was the Census Bureau, but the accuracy was less than that of direct allocations; assign $\delta = .20$. All other transfers were assigned upper and lower bounds in the same manner as the corresponding cell in the direct allocations matrix.

A.1.3.4 Margins

Transportation margins, by product type and mode, are obtained as totals and then prorated proportional to producers' prices across all purchasers of that commodity. Then margins in each input are summed for each purchaser and added to the directly allocated inputs. For all transport modes, $\delta = .25$ was assigned to the margins. Wholesale and retail trade margins may be expected to be more variable, and are sometimes computed as percentage markups over the already estimated transport margins. Therefore they are assigned $\delta = .35$.

A.1.4. CONCLUSIONS, APPLICATIONS, AND LIMITATIONS

Earlier work using maximum-upper-bound analyses had shown the dangers that might be encountered using results of input-output analyses.* Therefore, these estimates of uncertainty on the actual data were needed to check the maximum error bounds on the particular results we were interested in using (e.g., elements of the energy sector rows of the 1967 Leontief inverse matrix). It soon became evident that the magnitude of the uncertainties in

*See for example the results presented by Bullard and Sebald (1975).

the parameter estimation process that our maximum upper bound analysis would yield unsatisfactory results.

The above information is given to further illuminate the context in which these uncertainty estimates were made, and hopefully will discourage inappropriate applications of the results.

Finally, I repeat that the uncertainty estimates presented here are my own. I have listed many of the persons whom I interviewed, but they have not endorsed my interpretations of those interviews. If the absolute levels of the estimates are widely disputed (and I expect they will be) perhaps at least the relative levels will be accepted. On this basis we have performed stochastic error analyses on the 1967 U.S. input-output model for several cases; including doubling error margins presented here, to determine the sensitivity of the results to systematic bias in the estimates.

A.2 DIRECT ENERGY ALLOCATIONS

Knecht (1975) estimated error tolerances on all physical-unit energy transactions. These are coded in Table A.2-2, at the 90 sector level of detail, and in Tables A.2-3 and A.2-4 at the 101-sector level, and the codes ar explained in Table A.2-1 below.*

Table A.2-1

ENERGY TRANSACTION TOLERANCE CODES

Code	σ
00	$\mu=0$ and $3\sigma=10^{11}$ Btu)
01, 02,09,13	.05
04,41,16,18,19,20,24,28	.10
03,05,06,29,30	.15
07,12,14,15,17,22,23,26,27	.20
25	.25
08,10,11	.30
10	.35

* Note that instead of the 368-sector level of aggregation, the results presented here are consistent with the slightly aggregated 357-sector breakdown described by Bullard & Herendeen (1975). Dummy sectors consuming no energy have been deleted and public and private sectors producing the same primary product have been combined.

TABLE A.2-2

TOLERANCE CODES FOR DIRECT ENERGY USE DATA (90 SECTOR)

Sector Number	Sector Name	Energy Supplies Coal	Crude	Oil	Electric	Gas
1	COAL MINING	02	00	02	02	02
2	CRUDE PETROLEUM, GAS	11	02	02	02	02
3	REF'D PETROLEUM PROD'S	01	01	01	01	01
4	ELECTRIC UTILITIES	01	00	01	01	01
5	NATURAL GAS UTILITIES	11	01	11	11	01
6	LIVESTOCK...PRODUCTS	11	00	08	08	11
7	OTHER AGRIC'L PRODUCTS	11	00	08	08	11
8	FORESTRY AND FISHERY..	11	00	11	11	11
9	AG., FOR'Y...SERVICES	11	00	11	11	11
10	IRON...ORES MINING	11	00	02	02	02
11	NONFERROUS ORES MINING	11	00	02	02	02
12	STONE AND CLAY MINING.	02	00	02	02	02
13	CHEMICALS, ETC. MINING	11	00	02	02	02
14	NEW CONSTRUCTION	00	00	11	11	11
15	MAINT. AND REPAIR CON.	00	00	11	11	11
16	ORDNANCE AND ACCESSOR.	03	00	05	03	03
17	FOOD AND KINDRED PROD.	02	00	04	02	02
18	TOBACCO MANUFACTURING	02	00	04	02	02
19	FABRIC...THREAD MILLS	02	00	04	02	02
20	MISC. TEXTILE...FLOOR.	03	00	05	11	03
21	APPAREL	03	00	05	03	03
22	MISC. FAB. TEXTILE PRO	11	00	11	11	11
23	LUMBER...PROD'S, EXCEP	03	00	05	03	03
24	WOODEN CONTAINERS	11	00	05	03	11
25	HOUSEHOLD FURNITURE	03	00	05	03	03
26	OTHER FURNITURE AND...	03	00	05	11	03
27	PAPER AND...EXCEPT...	02	00	04	02	02
28	PAPERB'D CONTAINERS...	02	00	04	02	02
29	PRINTING AND PUBLISH'G	03	00	05	03	03
30	CHEMICALS AND...PROD'S	02	02	04	02	02
31	PLASTICS AND...MATER'S	02	00	04	02	02
32	DRUGS,...PREPARATIONS	02	00	04	02	02
33	PAINTS AND...PRODUCTS	02	00	04	02	02
34	PAVING MIXTURES AND...	11	00	06	02	02
35	ASPHALT FELTS AND COAT	02	00	06	02	02
36	RUBBER AND...PRODUCTS	02	00	04	02	02
37	LEATHER TANNING AND...	03	00	05	11	11
38	FOOTWEAR AND...PROD'S	03	00	05	03	03
39	GLASS AND GLASS PROD'S	02	00	04	02	02
40	STONE AND CLAY PROD'S	02	00	04	02	02
41	PRIM. IRON AND STEEL..	02	00	04	02	02
42	PRIM. NONFERROUS METAL	02	00	04	02	02
43	METAL CONTAINERS	02	00	06	02	02
44	HEAT., PLUMB...PROD'S	03	00	05	03	03
45	SCREW MACH. PROD'S,...	02	00	04	02	02
46	OTHER FAB. METAL PROD.	03	00	07	03	03
47	ENGINES AND TURBINES	02	00	04	02	02
48	FARM MACHINERY	02	00	04	02	02
49	CONSTRUCTION,...EQUIP.	03	00	05	03	03
50	MAT. HANDLING...EQUIP.	03	00	05	11	03

TABLE A.2-2 (continued)

Sector Number	Sector Name	Energy Supplies				
		Coal	Crude	Oil	Electric	Gas
51	METALWORKING...EQUIP'T	02	00	04	02	02
52	SPEC. INDUSTRY...EQUIP	03	00	05	11	03
53	GEN. INDUSTRIAL...EQ'T	02	00	04	02	02
54	MACHINE SHOP PRODUCTS	03	00	05	03	03
55	OFF., COMP'G...MACHINE	11	00	04	02	02
56	SERV. IND. MACHINES	03	00	05	11	03
57	ELEC. TRANS...APPARAT.	03	00	05	03	03
58	HOUSEHOLD APPLIANCES	03	00	05	11	03
59	ELEC. LIGHT'G...EQUIP.	03	00	05	03	03
60	RADIO, TV, COM. EQUIP.	03	00	05	03	03
61	ELEC. COMPONENTS...	02	00	04	02	02
62	MISC. ELEC...SUPPLIES	03	00	05	11	11
63	MOTOR VEHICHLES...EQ'T	02	00	04	02	02
64	AIRCRAFT AND PARTS	02	00	04	02	02
65	OTHER TRANS. EQUIPMENT	02	00	04	02	02
66	PROFESSIONAL...SUPPLIE	03	00	05	11	11
67	OPICAL...EQUIP. AND...	03	00	04	02	02
68	MISC. MANUFACTURING	03	00	05	03	11
69	RAILROADS AND...SERV'S	09	00	09	09	11
70	...HIGHWAY PASS. TRAN.	11	00	09	09	11
71	MOTOR FREIGHT TRANS...	11	00	09	11	11
72	WATER TRANSPORTATION	09	00	09	11	11
73	AIR TRANSPORTATION	11	00	09	11	11
74	PIPE LINE TRANSPORTA'N	11	00	11	11	09
75	TRANSPORTAION SERVICES	11	00	00	11	11
76	COM'NS EXCEPT RADIO...	11	00	11	11	11
77	RADIO AND TV BROADCAST	11	00	11	11	11
78	WATER AND SAN. SERV'S	11	00	11	11	11
79	WHOLESALE AND RETAIL..	11	00	11	11	11
80	FINANCE AND INSURANCE	11	00	11	12	11
81	REAL ESTATE AND RENTAL	00	00	11	11	11
82	HOTELS AND...EXCEPT...	11	00	11	11	51
83	BUSINESS SERVICES	11	00	11	12	11
84	AUTO. REPAIR AND SERV.	11	00	11	12	11
85	AMUSEMENTS	11	00	11	12	11
86	MED., ED. SERV'S AND..	11	00	11	11	11
87	FED. GOV'T ENTERPRISES	11	00	11	12	11
88	STATE AND LOCAL...EN'S	11	00	11	12	11
89	BUS. TRAV...AND GIFTS	00	00	00	00	00
90	OFFICE SUPPLIES	00	00	00	00	00
91	PERS. CONSUMP'N EXPEN.	11	00	01	12	01
92	GROSS...CAP. FORMATION	00	00	00	00	00
93	NET INVENTORY CHANGE	01	01	01	00	01
94	NET EXPORTS	01	01	01	01	01
95	FED. GOV'T...DEFENSE	11	00	11	11	11
96	FED. GOV'T...OTHER	11	00	11	11	11
97	STATE...GOV'T...EDUC'N	11	00	11	11	11
98	STATE...HEALTH,...SAN.	11	00	11	12	11
99	STATE...GOV'T...SAFETY	11	00	11	12	11
100	STATE...GOV'T...OTHER	11	00	11	12	11

TABLE A.2-3

TOLERANCE CODES FOR ENERGY END USES DATA (101 SECTOR)

Sector Number	Sector Name	Coke	Feed-Stocks	Mot. Pow.	Misc. Ther. Users	Water Heat	Space Heat	Air Cond.	Misc. Elec. Pow. Uses
1	COAL MINING	00	14	14	16	15	00	00	18
2	CRUDE PETROLEUM, GAS	00	14	14	16	15	00	00	18
3	GASIFIED COAL	00	21	21	21	21	21	21	21
4	REF'D PETROLEUM PROD'S	00	13	14	19	15	14	17	20
5	NATURAL GAS UTILITIES	00	14	14	00	14	22	14	23
6	FOSSIL ELECTRIC UTIL'S	00	14	14	00	14	22	14	23
7	NUCLEAR ELEC. UTIL'S	00	14	14	00	14	22	14	23
8	RENEWABLE ELEC. UTIL'S	00	14	14	00	14	22	14	23
9	ORE-REDUC. FEEDSTOCKS	00	00	00	00	00	00	00	00
10	CHEMICAL FEEDSTOCKS	00	00	00	00	00	00	00	00
11	MOTIVE POWER	00	00	00	00	00	00	00	00
12	MISC. THERMAL USES	00	00	00	00	00	00	00	00
13	WATER HEAT	00	00	00	00	00	00	00	00
14	SPACE HEAT	00	00	00	00	00	00	00	00
15	AIR-CONDITIONING	00	00	00	00	00	00	00	00
16	MISC. ELEC. POWER USES	00	00	00	00	00	00	00	00
17	LIVESTOCK...PRODUCTS	00	14	14	00	14	22	00	23
18	OTHER AGRIC'L PRODUCTS	00	13	13	00	14	22	00	23
19	FORESTRY AND FISHERY..	00	14	14	00	14	22	00	23
20	AG., FOR'Y...SERVICES	00	14	14	00	14	22	00	23
21	IRON...ORES MINING	00	14	14	16	15	00	00	18
22	NONFERROUS ORES MINING	00	14	14	16	15	00	00	18
23	STONE AND CLAY MINING.	00	14	14	16	15	00	00	18
24	CHEMICALS, ETC. MINING	00	14	14	16	15	00	00	18
25	NEW CONSTRUCTION	00	24	14	25	25	25	00	23
26	MAINT. AND REPAIR CON.	00	24	14	25	25	00	00	23
27	ORDNANCE AND ACCESSOR.	00	14	14	29	15	14	17	30
28	FOOD AND KINDRED PROD.	13	14	14	19	15	14	17	20
29	TOBACCO MANUFACTURING	00	14	14	19	15	14	17	20
30	FABRIC...THREAD MILLS	00	14	14	19	15	14	17	20
31	MISC. TEXTILE...FLOOR.	00	14	14	29	15	14	17	30
32	APPAREL	00	14	14	29	15	14	17	30
33	MISC. FAB. TEXTILE PRO	00	14	14	29	15	14	17	30
34	LUMBER...PROD'S, EXCEP	00	14	14	29	15	14	17	30
35	WOODEN CONTAINERS	00	14	14	29	15	14	00	30
36	HOUSEHOLD FURNITURE	00	14	14	29	15	14	00	30
37	OTHER FURNITURE AND...	00	14	14	29	15	14	00	30
38	PAPER AND...EXCEPT...	00	13	14	19	15	14	17	20
39	PAPERB'D CONTAINERS...	00	14	14	19	15	14	17	20
40	PRINTING AND PUBLISH'G	00	14	14	29	15	14	17	30
41	CHEMICALS AND...PROD'S	00	13	14	19	15	14	17	20
42	PLASTICS AND...MATER'S	00	13	14	19	15	14	17	20
43	DRUGS,...PREPARATIONS	00	14	14	19	15	14	17	20
44	PAINTS AND...PRODUCTS	00	13	14	19	15	14	17	20
45	PAVING MIXTURES AND...	00	13	14	19	15	14	00	20
46	ASPHALT FELTS AND COAT	00	13	14	19	15	14	00	20
47	RUBBER AND...PRODUCTS	00	14	14	19	15	14	17	20
48	LEATHER TANNING AND...	00	14	14	29	15	14	00	30
49	FOOTWEAR AND...PROD'S	00	14	14	29	15	14	00	30
50	GLASS AND GLASS PROD'S	00	14	14	19	15	14	17	20
51	STONE AND CLAY PROD'S	13	14	14	19	15	14	17	20
52	PRIM. IRON AND STEEL..	13	14	14	19	15	14	17	20
53	PRIM. NONFERROUS METAL	13	14	14	19	15	14	17	20

TABLE A.2-3 (continued)

Sector Number	Sector Name	Coke	Feed-Stocks	Mot. Pow.	Misc. Ther. Uses.	Water Heat	Space Heat	Air-Cond.	Misc. Elec. Pow. Uses.
54	METAL CONTAINERS	00	14	14	19	15	14	00	20
55	HEAT., PLUMB...PROD'S	13	14	14	29	15	14	17	30
56	SCREW MACH. PROD'S,...	00	14	14	19	15	14	17	20
57	OTHER FAB. METAL PROD.	13	14	14	29	15	14	17	30
58	ENGINES AND TURBINES	13	14	14	19	15	14	17	20
59	FARM MACHINERY	13	14	14	19	15	14	17	20
60	CONSTRUCTION,...EQUIP.	13	14	14	29	15	14	17	30
61	MAT. HANDLING...EQUIP.	00	14	14	29	15	14	17	30
62	METALWORKING...EQUIP'T	00	14	14	19	15	14	17	20
63	SPEC. INDUSTRY...EQUIP	13	14	14	29	15	14	17	30
64	GEN. INDUSTRIAL...EQ'T	13	14	14	19	15	14	17	20
65	MACHINE SHOP PRODUCTS	13	14	14	29	15	14	17	30
66	OFF., COMP'G...MACHINE	00	14	14	00	15	14	17	20
67	SERV. IND. MACHINES	00	14	14	29	15	14	17	30
68	ELEC. TRANS...APPARAT.	00	14	14	29	15	14	17	30
69	HOUSEHOLD APPLIANCES	00	14	14	29	15	14	17	30
70	ELEC. LIGHT'G...EQUIP.	13	14	14	29	15	14	17	30
71	RADIO, TV, COM. EQUIP.	00	14	14	29	15	14	17	30
72	ELEC. COMPONENTS...	00	14	14	19	15	14	17	20
73	MISC. ELEC...SUPPLIES	00	14	14	29	15	14	17	30
74	MOTOR VEHICHLES...EQ'T	13	14	14	19	15	14	17	20
75	AIRCRAFT AND PARTS	00	14	14	19	15	14	17	20
76	OTHER TRANS. EQUIPMENT	00	14	14	19	15	14	17	20
77	PROFESSIONAL...SUPPLIE	00	14	14	29	15	14	17	30
78	OPICAL...EQUIP. AND...	00	14	14	19	15	14	17	20
79	MISC. MANUFACTURING	00	14	14	29	15	14	17	30
80	RAILROADS AND...SERV'S	00	13	13	00	15	22	00	18
81	...HIGHWAY PASS. TRAN.	00	13	13	00	15	22	00	18
82	MOTOR FREIGHT TRANS...	00	13	13	00	15	22	00	18
83	WATER TRANSPORTATION	00	13	13	00	15	22	00	18
84	AIR TRANSPORTATION	00	13	13	00	15	22	00	18
85	PIPE LINE TRANSPORTA'N	00	22	22	00	15	22	00	18
86	TRANSPORTAION SERVICES	00	00	00	00	15	22	00	18
87	COM'NS EXCEPT RADIO...	00	27	27	00	14	22	14	23
88	RADIO AND TV BROADCAST	00	27	27	00	14	22	14	23
89	WATER AND SAN. SERV'S	00	27	27	00	14	22	14	23
90	WHOLESALE AND RETAIL..	00	27	27	26	26	22	14	23
91	FINANCE AND INSURANCE	00	27	27	00	14	22	14	23
92	REAL ESTATE AND RENTAL	00	27	27	00	14	22	14	23
93	HOTELS AND...EXCEPT...	00	27	27	26	52	52	14	23
94	BUSINESS SERVICES	00	27	27	00	14	22	14	23
95	AUTO. REPAIR AND SERV.	00	27	27	00	14	22	14	23
96	AMUSEMENTS	00	27	27	26	26	22	14	23
97	MED., ED. SERV'S AND..	00	27	27	26	26	22	14	23
98	FED. GOV'T ENTERPRISES	00	27	27	00	14	22	14	23
99	STATE AND LOCAL...EN'S	00	27	27	00	14	22	14	23
100	BUS. TRAV...AND GIFTS	00	00	00	00	00	00	00	00
101	OFFICE SUPPLIES	00	00	00	00	00	00	00	00
102	PERS. CONSUMP'N EXPEN.	00	28	28	28	28	28	28	28
103	GROSS...CAP. FORMATION	00	00	00	00	00	00	00	00
104	NET INVENTORY CHANGE	00	00	00	00	00	00	00	00
105	NET EXPORTS	00	00	00	00	00	00	00	00
106	FED. GOV'T...DEFENSE	00	13	13	26	26	22	14	23
107	FED. GOV'T...OTHER	00	27	27	00	14	22	14	23
108	STATE...GOV'T...EDUC'N	00	27	27	26	26	22	14	23
109	STATE...HEALTH,...SAN.	00	27	27	26	26	22	14	23
110	STATE...GOV'T...SAFETY	00	27	27	00	14	22	14	23
111	STATE...GOV'T...OTHER	00	27	27	00	14	22	14	23

TABLE A.2-4

TOLERANCE CODES FOR TRANSACTIONS FROM ENERGY SUPPLY SECTORS (101 SECTOR)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	02	11	10	01	11	01	00	00	13	13	00	13	00	13	00	00
2	00	02	00	01	01	00	00	00	00	00	00	00	00	00	00	00
3	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00	00
4	02	02	00	01	11	01	00	00	00	13	13	13	13	13	00	00
5	02	02	10	01	01	01	00	00	00	13	00	13	13	13	13	00
6	02	02	00	01	11	01	00	00	00	00	00	13	13	13	13	13
7	02	02	00	01	11	00	01	00	00	00	00	13	13	13	13	13
8	02	02	00	01	11	00	00	01	00	00	00	13	13	13	13	13

Notes: Refer to Table A.2-3 for sector names.

A.3 DISPERSION FACTORS FOR SMALL-MAGNITUDE FIGURES

Often the uncertainty on a column of figures, χ_i would be described in terms of "a dispersion factor D_1 for $\chi_i = B$ increasing to a dispersion factor D_2 for the smallest value reported." Taking this lower bound to be $\chi_i = A$ where $A = \$10^5$ for the 1967 U.S. input-output tables, and assuming a linear dependence of $D(\chi)$ on $\log(\chi)$ we obtain the following expression for D as a function of χ_i. Let $D(\chi) = a \log(\chi) + b$ where $a = (D_2 - D_1)/(\log A - \log B)$ and $b = (D_1 \log A - D_2 \log B)/(\log A - \log B)$. It is easy to verify that $D(\chi)$ takes values D_1 and D_2 at $\chi = B$ and $\chi = A$ respectively.

Obviously this is a crude approximation, but it actually may be even too refined when viewed from the perspective of the person estimating the uncertainty.

A.4 PROBABLE VALUES OF "ZERO" ELEMENTS

Since few transactions can be defined to be zero, the published figures truncated at $\$10^5$ dollars may be misleading. It is probably true that if we examined in detail the transactions of all firms in the U.S. defined by a particular transaction cell in the I-O table, we would find at least one nonzero transaction. Therefore the following approximation was used to estimate the probable distribution of nonzero values between the lower and upper bounds $[0, \$10^5]$.

Let X be the absolute value of a normal random variable Y with mean 0 and $\sigma = 10^5/3$. Then X takes nearly all its values between 0 and 10^5. By truncating X at 10^5, in the sense that larger values are discarded and resampled, the resulting random variable takes all of its values in $[0, 10^5]$ with the great bulk of its unit probability accumulated near zero.

For direct energy transactions, the cutoff was 10^{11}Btu, which corresponds to approximately the same dollar value.

Details of the "folded normal" distribution are given in Appendix C.

TABLE B-1. 30 Sector Model

30 SECTOR MODEL	BEA SECTOR	
1.	7	Coal
2.	8	Crude Oil and Natural Gas
3.	31.01	Refined Petroleum Products
4.	68.01	Electric Utilities
5.	68.02	Natural Gas Utilities
6.	1-4	Agriculture
7.	5-10	Mining
8.	11-12	Construction
9.	14, 15, 29	Food and Drugs
10.	16-19	Textiles and Apparel
11.	20-26	Wood and Paper Products
12.	27, 28, 30-32	Paint, Plastics and Oil Products
13.	33, 34	Leather and Shoes
14.	35, 36	Stone, Clay and Glass Products
15.	37-42	Metals and Metal Products
16.	43-52	Machinery
17.	53-58	Electrical Equipment and Appliances
18.	59-61	Cars, Planes and Transport Equipment
19.	62-64, 13	Miscellaneous Manufacturing
20.	65.01	Rail Transport
21.	65.02	Local Passenger Transport
22.	65.03	Truck Transport and Warehousing
23.	65.04	Water Transport
24.	65.05	Air Transport
25.	65.06	Pipeline Transport
26.	66-67	Radio, TV, Communications
27.	69	Wholesale and Retail Trade
28.	70-71	Finance, Insurance and Real Estate
29.	65.07, 68.03, 72-79	Services
30.	81-82	Business Travel and Office Supplies

Sequence No.	Code		Sector
1	7	Coal mining	I
2	8	Crude petroleum and natural gas	I
-	-	Converted Coal	-
3	31.01	Petroleum refining and related products	I
5	68.02	Gas utilities	C
4	68.01	Fossil electric utilities	C
	68.01	Nuclear electric utilities	
	68.01	Renewable electric utilities	
-	-	Coke (ore-reduction) feedstocks	-
-	-	Other feedstocks	-
-	-	Motive power	-
-	-	Miscellaneous thermal	-
-	-	Water heat	-
-	-	Space heat	-
-	-	Air conditioning	-
-	-	Electric power	-
6	1	Livestock and livestock products	C
7	2	Other agricultural products	C
8	3	Forestry and fishery products	C
9	4	Agricultural, forestry and fishery services	C
10	5	Iron and ferroalloy ores mining	I
11	6	Nonferrous metal ores mining	I
12	9	Stone and clay mining and quarrying	I
13	10	Chemicals and fertilizer mineral mining	I
14	11	New construction	C
15	12	Maintenance and repair construction	C
16	13	Ordnance and accessories	I
17	14	Food and kindred products	I
18	15	Tobacco manufactures	I
19	16	Broad and narrow fabrics, yarn and thread mills	I
20	17	Miscellaneous textile goods and floor coverings	I
21	18	Apparel	I
22	19	Miscellaneous fabricated textile products	I
23	20	Lumber and wood products, except containers	I
24	21	Wooden containers	I
25	22	Household furniture	I
26	23	Other furniture and fixtures	I
27	24	Paper and allied products except containers and boxes	I
28	25	Paperboard containers and boxes	I
29	26	Printing and publishing	I
30	27	Chemicals and selected chemical products	I
31	28	Plastics and synthetic materials	I
32	29	Drugs, cleaning and toilet preparation	I
33	30	Paints and allied products	I
34	31.02	Paving mixtures and blocks	I
35	31.03	Asphalt felts and coatings	I
36	32	Rubber and miscellaneous plastics products	I
37	33	Leather tanning and industrial leather products	I
38	34	Footwear and other leather products	I
39	35	Glass and glass products	I
40	36	Stone and clay products	I
41	37	Primary iron and steel manufacturing	I
42	38	Primary nonferrous metals manufacturing	I
43	39	Metal containers	I
44	40	Heating, plumbing and fabricated structural metal products	I
45	41	Screw machine prods., bolts, nuts, etc. & metal stampings	I

No.	Sequence No.	Code	
57	46	42	Other fabricated metal products
58	47	43	Engines and turbines
59	48	44	Farm machinery
60	49	45	Construction, mining, oil field machinery, equipment
61	50	46	Materials handling machinery and equipment
62	51	47	Metalworking machinery and equipment
63	52	48	Special industry machinery and equipment
64	53	49	General industrial machinery and equipment
65	54	50	Machine shop products
66	55	51	Office, computing and accounting machines
67	56	52	Service industry machines
68	57	53	Elec. transmission & dist. equip. & elec. indus. apparatus
69	58	54	Household appliances
70	59	55	Electric lighting and wiring equipment
71	60	56	Radio, television and communication equipment
72	61	57	Electronic components and accessories
73	62	58	Miscellaneous electrical machinery, equipment and supplies
74	63	59	Motor vehicles and equipment
75	64	60	Aircraft and parts
76	65	61	Other transportation equipment
77	66	62	Professional, scientific & controlling instruments & supp.s
78	67	63	Optical, ophthalmic, & photographic equipment & supplies
79	68	64	Miscellaneous manufacturing
80	69	65.01	Railroads and related services
81	70	65.02	Local, suburban & interurban highway passenger transpor.
82	71	65.03	Motor freight transportation and warehousing
83	72	65.04	Water transportation
84	73	65.05	Air transportation
85	74	65.06	Pipe line transportation
86	75	65.07	Transportation services
87	76	66	Communications except radio and television broadcasting
88	77	67	Radio and TV broadcasting
89	78	68.03	Water and sanitary services
90	79	69	Wholesale and retail trade
91	80	70	Finance and insurance
92	81	71	Real estate and rental
93	82	72	Hotels & lodging; pers. & repair serv., except auto repair
94	83	73	Business services
95	84	75	Automobile repair and services
96	85	76	Amusements
97	86	77	Medical, educational serv. and nonprofit organizations
98	87	78	Federal government enterprises
99	88	79	State and local government enterprises
100	89	81	Business travel, entertainment and gifts
101	90	82	Office supplies
102	91	96.60	Personal consumption expenditures
103	92	96.70	Gross private fixed capital formation
104	93	96.80	Net inventory change
105	94	96.90	Net exports
106	95	97.10	Federal government purchases, defense
107	96	97.20	Federal government purchases, other
108	97	98.60	State and local government purchases, education
109	98	98.70	State and local government purchases, health, welf. & sanit.
110	99	98.80	State and local government purchases, safety
111	100	98.90	State and local government purchases, other

TABLE B-2. 101 Sector Model

Appendix C. Treatment of Zero Values

In section 1 the variance of a "folded" normal is computed while section 2 details the relationship between the variance of a lognormal and its dispersion factor.

C.1 Variance of a "Folded" Normal Distribution

The variance of any random variable, X, is given by $\mathrm{Var}(X) = E(X^2) - (E(X))^2$, where E denotes expected value. If Y is $N(0,1)$ then $1 = \mathrm{Var}\,(Y) = E(Y^2) - 0^2$ so $E(Y^2) = 1$. Z is said to be "folded" $N(0,1)$, if $Z = \mathrm{ABS}(Y)$. Then $E(Z^2) = E(Y^2) = 1$. Therefore $\mathrm{Var}\,(Z) = E(Z^2) - (E(Z))^2$. But

$$E(Z) = \frac{2}{\sqrt{2\Pi}} \int_0^\infty t\, e^{-\frac{t^2}{2}}\, d_t = \frac{2}{\sqrt{2\Pi}} \left. -e^{-\frac{t^2}{2}} \right|_0^\infty = \frac{2}{\sqrt{2\Pi}}$$

therefore $\mathrm{Var}\,(Z) = 1 - (E(Z))^2 = 1 - \frac{2}{\Pi}$

If we fold a normal random variable with three sigmas equal to b then the variance computed above is multiplied by a factor of $(\frac{b}{3})^2$. Conversely if we know the variance V of a folded normal then one sigma of the underlying normal is $\sqrt{\dfrac{V}{1-\frac{2}{\Pi}}}$

C.2 Variance of a Lognormal Distribution

If a given cell is lognormal with published mean M and three sigma dispersion factor D, then we sample for this cell by exponentiating a $N(\alpha,\beta^2)$ random variable where $\beta = \frac{\ln D}{3}$ and $\alpha = \ln M - \frac{\ln^2 D}{18}$.

The variance of a lognormal random variable with parameters α and β is given by $V = e^{2\alpha+\beta^2} (e^{\beta^2} -1) = M^2 (e^{\beta^2} -1)$. Substituting $\frac{\ln D}{3}$ for β we obtain $V = M^2 * \exp \frac{\ln^2 D}{9} -1$. Conversely, we can solve this equation for D:

$$D = \exp (3* \ln(1+V/M^2))$$

Incidentally, at the changeover point from normal to lognormal we have $ABS\left(\frac{3\sqrt{V}}{M}\right) = .4$ or $\frac{V}{M^2} = .0178$. At this point D = 1.489. As we cross the changeover point from normal to lognormal we switch from a normal with range between 60% and 140% of the published mean value to a lognormal with D = .1489 and 1/D = .67.

APPENDIX D. RANDOM NUMBER GENERATOR VERIFICATION

By Robert Bohrer and Dan Putnam

D.1 INTRODUCTION

The present Monte Carlo study of the sensitivity of Input-Output data to stochastic estimation errors requires the use of a fast, reliable normal random number generator. In the present case each Monte Carlo trial requires roughly ten thousand random numbers to perturb the parameters in the Input-Output model. Since several hundred trials are required to obtain useful results, speed is an important consideration in the choice of a generator for this application. Furthermore, a necessary condition for the validity of the results is, of course, that the random inputs conform to the modeling assumptions. In this case the required inputs are independent, normally distributed random numbers.

The generators in the International Mathematical and Statistical Libraries* were given special consideration for use in the Monte Carlo study because of the good reputation of IMSL and the availability of IMSL at most large IBM installations. One generator, GGNRF, was especially appealing since it was designed specifically as a fast normal random number generator and had already been optimized and coded in assembler language for efficiency. The one flaw in the qualifications of GGNRF was that the existing documentation of its statistical properties was inadequate for our needs. Although the distributional properties had been checked with several goodness of fit tests, independence had been

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* Available from International Mathematical and Statistical Libraries (IMSL) Inc., Sixth Floor, GNB Building, 7500 Bellaire Boulevard, Houston, Texas 77036.

checked only up to five lags. Documentation of these tests may be found in Kuki (1974). While it is impossible to test all aspects of randomness, it is prudent to examine at least those aspects most important to the particular application.* The present Monte Carlo study requires independence among the inputs to each Monte Carlo trial and among the separate trials as well. The tests described in this paper were designed by the first author to examine both types of independence. Tests of normality were also included for the sake of completeness.

These properties may be tested by selecting several seeds for the generator and examining three sequences obtained from each. The first two sequences are chosen so that in the actual simulation they would occupy corresponding positions in the inputs to consecutive simulation runs. Thus, the first N numbers drawn from a seed constitute the first sequence. Then, as many more numbers are generated as would be needed to complete one simulation run. The second sequence then consists of the next N numbers drawn. To examine the independence between these first two sequences, a third sequence of 2N numbers is formed by shuffling the other two sequences so that the odd numbered entries are taken in order from the first sequence and the even numbered entries are taken in order from the second sequence.

It is desirable to use tests which are sensitive to as many departures from the hypotheses as possible. The methods used here allow tests of the independence of $X(t)$ and $X(t+L)$ for all lags L less than

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*For a discussion of random number generation and testing see Jansson (1966).

or equal to the sample size N. For this reason and because the statistical power of these tests increases with sample size, the value of N used in defining the sequences above should be as large as possible. While it would be desirable to make N as large as the number of inputs to a Monte Carlo trial, this possibility was precluded by considerations of the availability of computing resources in the statistical analyses of the random number samples. In this study N = 1024 was chosen with these considerations in mind and because of the efficiency of working with a power of 2 in Fast Fourier Transformation. Five seeds were selected for the tests that follow, again with this number being determined partly by considerations of available computing resources versus the quantity of information generated.

Section 2 details the tests performed on the shuffled sequences obtained from the five seeds to check independence between Monte Carlo trials. Section 3 describes the tests on the two unshuffled sequences from each seed to examine the independence of the inputs to a single trial. Finally, section 4 documents the test performed on the unshuffled sequences to check that the samples are normally distributed with mean zero and variance one.

D.2 INDEPENDENCE BETWEEN MONTE CARLO SAMPLES

To test the independence of consecutive runs from the Monte Carlo experiment, the shuffled sequences from the five seeds were examined. Given the hypothesis of independence between simulation runs, the sample autocovariances at odd lags, L, of the shuffled sequences should be independent and standard normal (i.e., N(0,1)) when multiplied by $\sqrt{2048-L}$.* A test of the independence of consecutive Monte Carlo trials may then be made by using the Kolmogrov-Smirnov (K-S) statistic to compare the sample distribution function of the adjusted autocovariances at odd lags to the standard normal distribution function. The sample used for this test was therefore $S_1 \cdots S_{256}$, $S_i = AC_{2i-1} * \sqrt{2048-(2i-1)}$ where the AC_{2i-1} are the odd lag autocovariances. Table 1 lists the five K-S statistics and P-levels obtained from the five shuffled sequences in this way. The tabled statistics represent the square root of the sample

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*That the mean is 0 and the variance is 1/2048-L follows from simple calculation with expectations. The asymptotic normality and independence follow from careful application of a multivariate central limit theorem such as theorem 9.2.3 in Wilks (1962). For example, normality of the lag L covariance estimate is shown by applying the theorem to the two sequences

$$(X_1 * X_{L+1}, \cdots X_L * X_{2L}, X_{2L+1} * X_{3L+1}, \cdots)$$

and $$(X_{L+1} * X_{2L+1}, \cdots X_{2L} * X_{3L}, X_{3L+1} * X_{4L+1}, \cdots).$$

If the covariance estimates are adjusted for the sample mean, then the tests for the hypothesized means and covariances can be made separately, each having valid significance level, even under certain natural discrepancies from the other hypothesis. Also, a theorem in section 20.6 of Cramér (1946) shows that the large sample distribution theory is exactly the same as described above.

size times the maximum absolute difference between sample and hypothesized distribution functions. The P-level is defined as the probability that a truly normal sample would have achieved a larger value for the K-S statistic than the value actually observed. Given the independence hypothesis, the P-levels should be independent and uniformly distributed on the unit interval. This fact allows the use of the following summary statistic: if $P_1 \cdots P_n$ are independent and uniform on the unit interval, then $-2*\sum_1^n \ln(P_i)$ is chi square with 2n degrees of freedom.* This sample statistic and its own P-level are included in Table 1 along with the K-S statistics and their P-levels as an additional check and summary.

TABLE 1

	Statistic	P-level
1	.744	.64
2	.869	.44
3	.548	.92
4	.807	.53
5	.772	.59
$-2*\Sigma \ln(P_i)$	5.03	.89

*This derives from the fact that if P is uniform on [0,1], then $-2*\ln(P)$ is exponentially distributed with distribution function $1-e^{-X/2}$. The result now follows by verifying that the associated density function is that of a chi-square random variable with two degrees of freedom.

A plot of the sample distribution function with the worst fit (from seed #2 in this case) is shown in Figure 1. Even in this worst case note how closely the sample distribution function fits the hypothesized distribution.

The K-S tests described above provide a check on independence in the time domain; a check in the frequency domain was also performed. By summing the Fast Fourier Transform of each of the shuffled sequences, sample integrated periodograms were obtained. Under the independence hypothesis, the integrated periodogram values should increase linearly from zero to one as the frequencies increase from zero to one-half. The Grenander-Rosenblatt (G-R) statistic may be used to measure the discrepancy of the sample integrated periodogram from linearity. If the hypothesis is true then a factor of $\sqrt{2048} / \sqrt{2} = 32$ times the maximum absolute difference between the sample integrated periodogram and twice the corresponding frequency should have the distribution calculated in Hannan (1967); departures from independence will tend to make the sample statistics too large to fit the distribution. The five sample statistics and corresponding P-levels are shown in Table 2 below along with the chi-square summary statistic defined in the last section. Seed #1 had the worst P-level so a graph of the corresponding sample integrated periodogram* is included in Figure 2.

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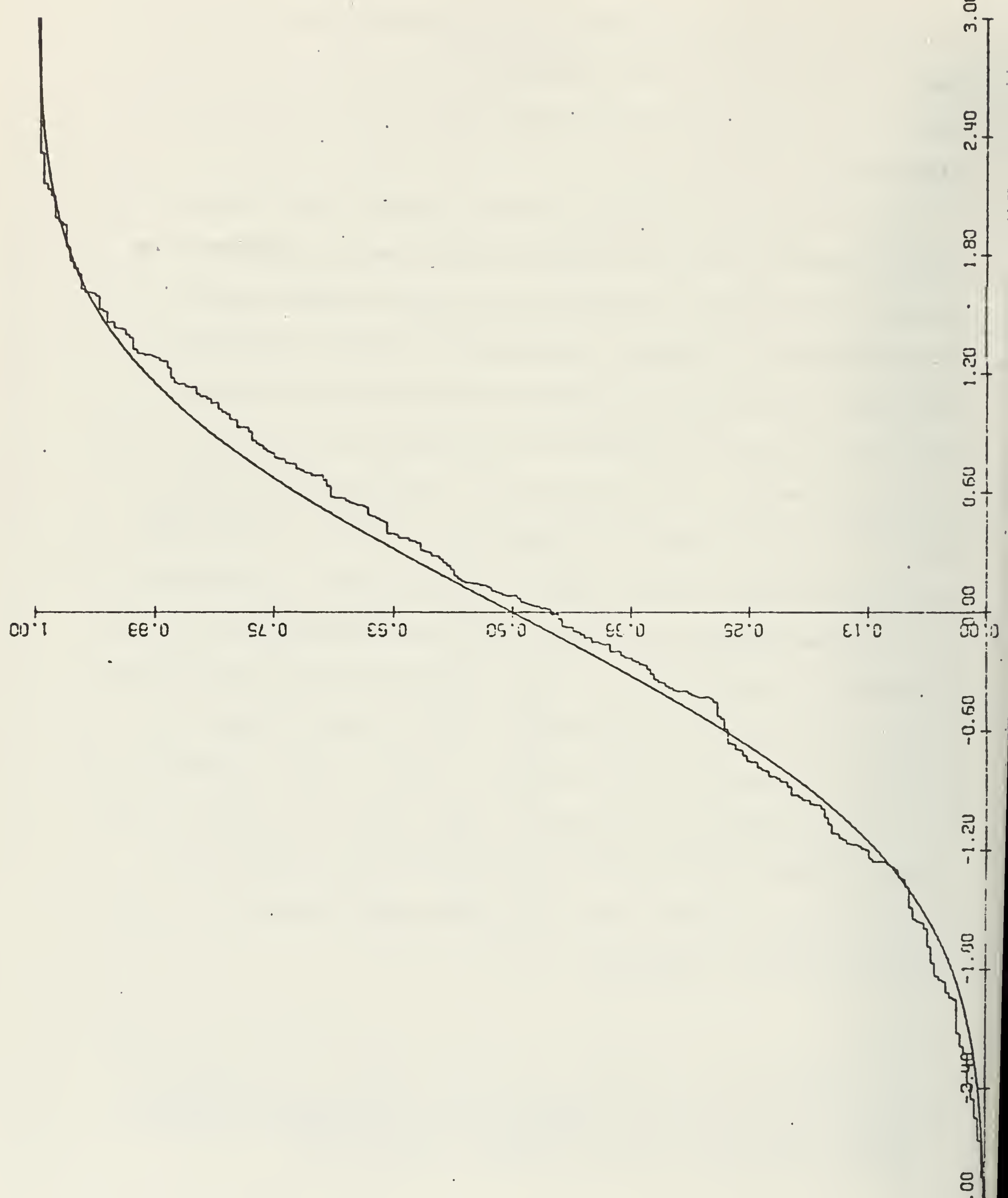
*Computed with SOUPAC program FASPER available from Computing Services Office, University of Illinois, Urbana, Illinois 61801.



1.00
0.88
0.75
0.63
0.50
0.38
0.25
0.13
0.00
-3.00
-2.40
-1.80
-1.20
-0.60
0.00
0.60
1.20
1.80
2.40
3.00

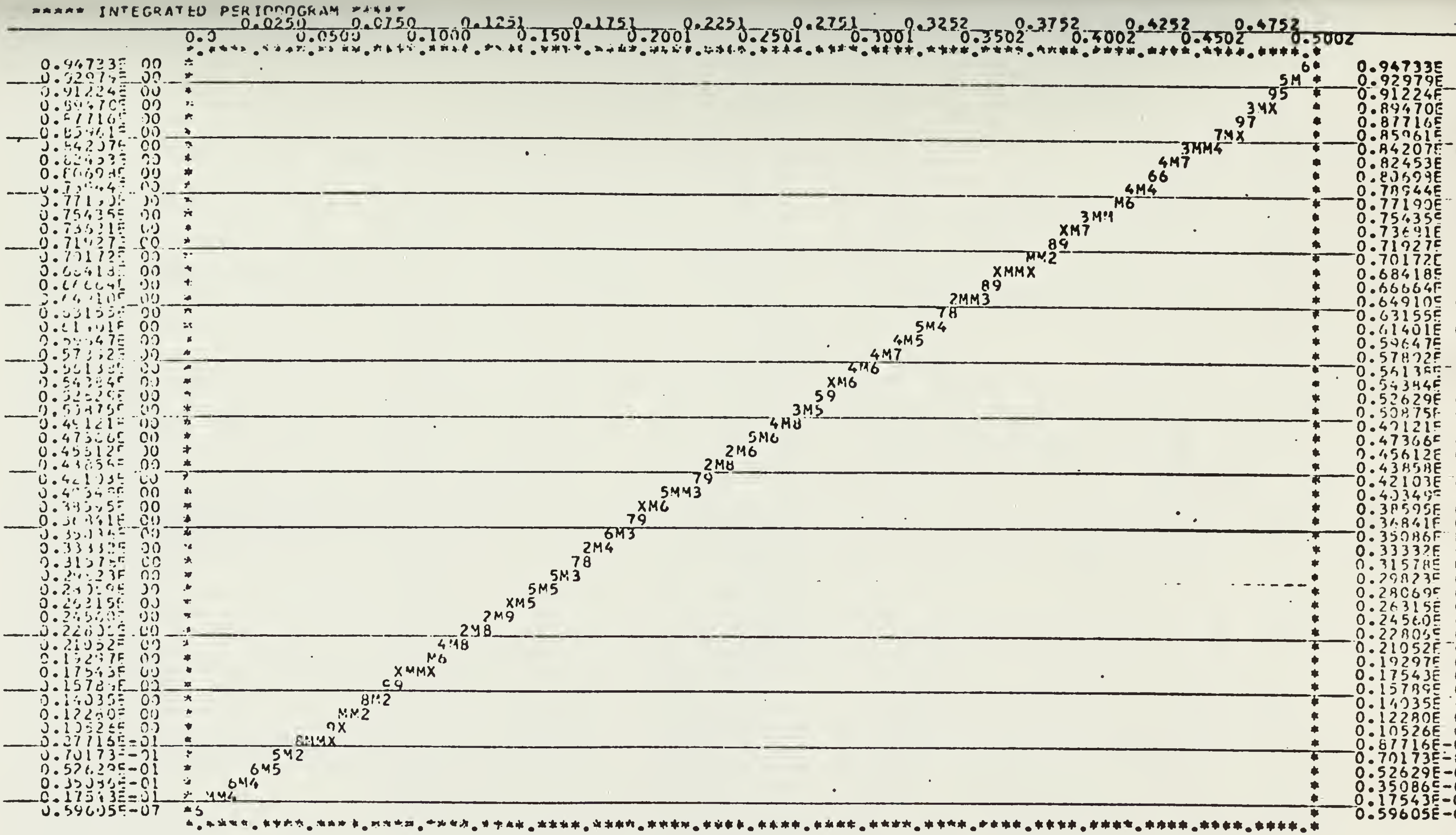


Figure 2. Integrated periodogram output for shuffled sequence drawn from seed 1.

TABLE 2

	Statistic	P-level
1	2.243	.04
2	.725	.88
3	1.578	.33
4	1.004	.63
5	.671	.92
$-2*\Sigma \ln(P_i)$	10.00	.44

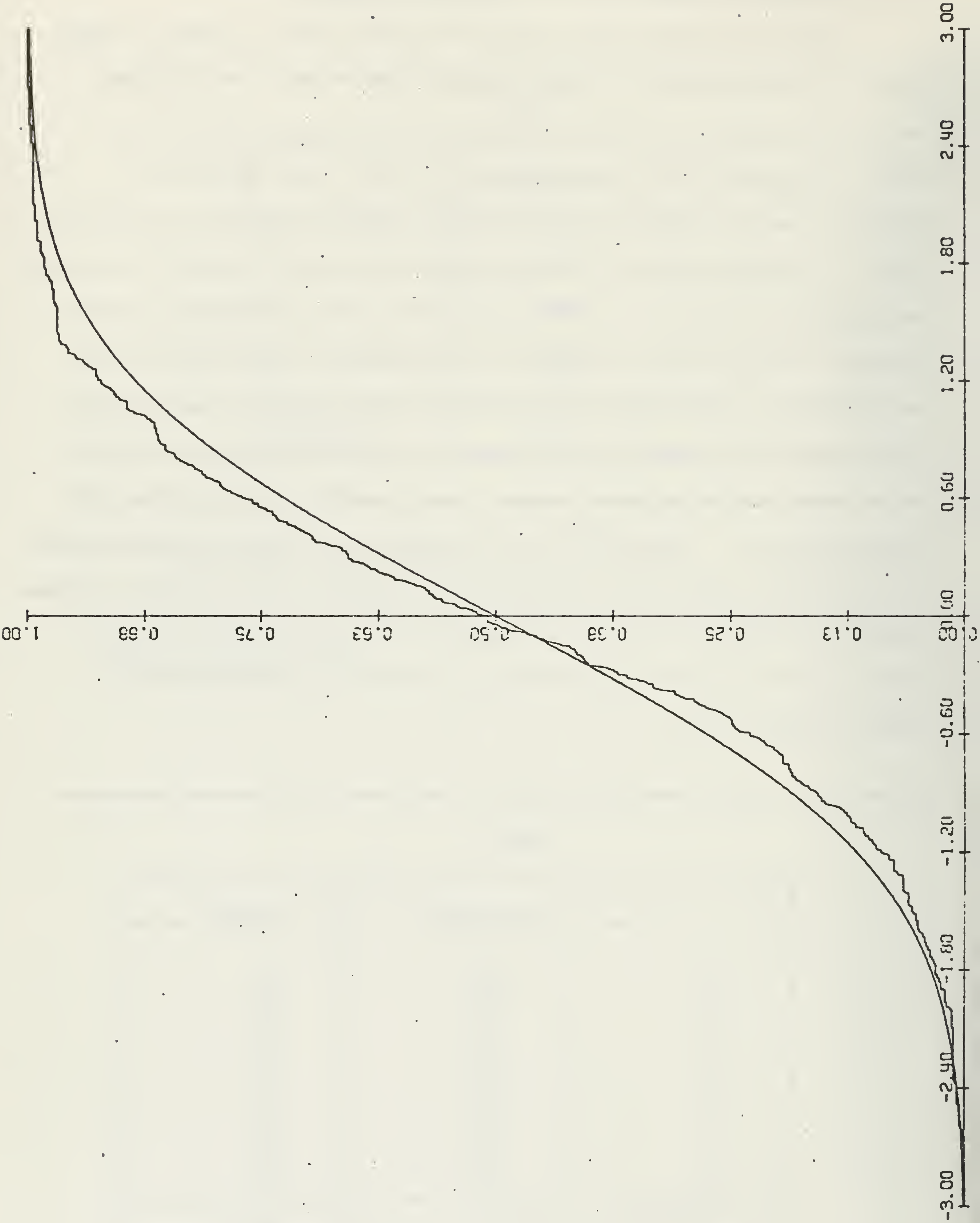
The results of this series of tests, like those of the preceding section, are quite satisfactory and give no cause to suspect interdependence between simulation runs.

D.3 INDEPENDENCE BETWEEN INPUTS TO A MONTE CARLO SAMPLE

While the frequency domain tests of the last section actually include a test of the independence of sample inputs, further tests were performed and are described below. The next two series of tests paralleled the two series of the last section in methodology, but this time attention was focused on the two sequences of 1024 numbers generated from each of the five seeds. In the time domain, the autocovariances at lags L = 1,512 of a sequence were adjusted by a factor of $\sqrt{1024-L}$ and tested with the Kolmogorov-Smirnov statistic. Again, under the hypothesis of independence within each of the ten sequences of 1024 numbers, the adjusted autocovariances of each sequence should constitute a standard normal sample. The resulting sample statistics and P-levels along with the chi-square summary statistic and its P-level are shown in Table 3. Similarly, the ten sequences were tested for independence in the frequency domain with the Fast Fourier Transform just as in the previous section. The sample statistics and summary statistic are shown in Table 4. Again, the worst cases are illustrated for the time and frequency domain tests in Figures 3 and 4.

TABLE 3

	Statistic	P-level
1A	1.132	.15
1B	1.214	.11
2A	1.077	.20
2B	.573	.90
3A	.882	.42
3B	1.106	.17
4A	.745	.64
4B	.657	.78
5A	.645	.80
5B	.491	.97
$-2^*\Sigma \ln(P_i)$	18.81	.53


1.00
0.88
0.75
0.63
0.50
0.38
0.25
0.13
0.00
-3.00
-2.40
-1.80
-1.20
-0.60
0.00
0.60
1.20
1.80
2.40
3.00

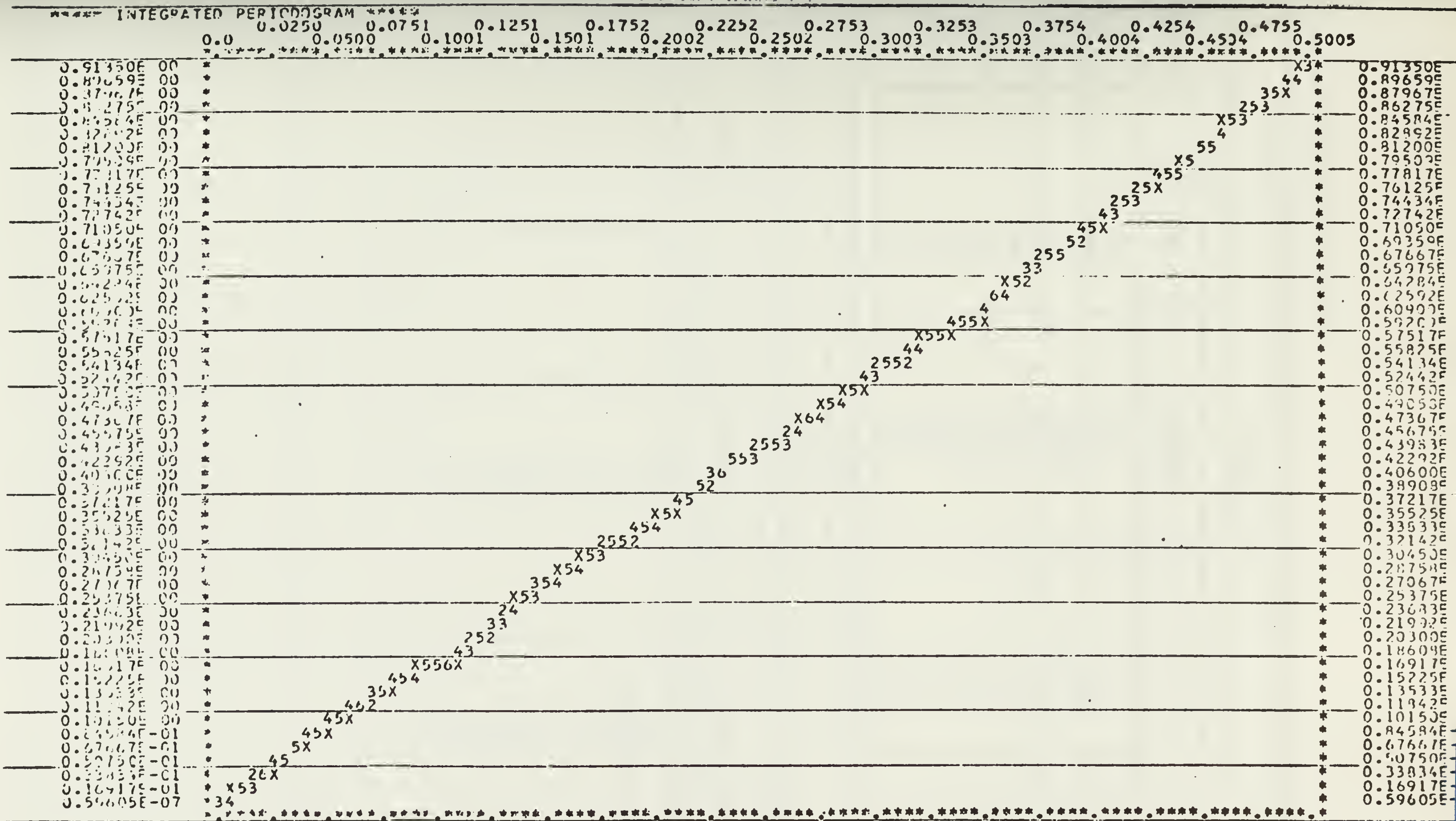


Figure 4. Integrated periodogram output for first sequence drawn from seed 2.

TABLE 4

	Statistic	P-level
1A	.659	.93
1B	2.259	.05
2A	2.439	.03
2B	1.461	.29
3A	.602	.96
3B	1.524	.25
4A	1.423	.31
4B	.910	.71
5A	.866	.75
5B	.833	.79
$-2*\Sigma \ln(P_i)$	22.55	.31

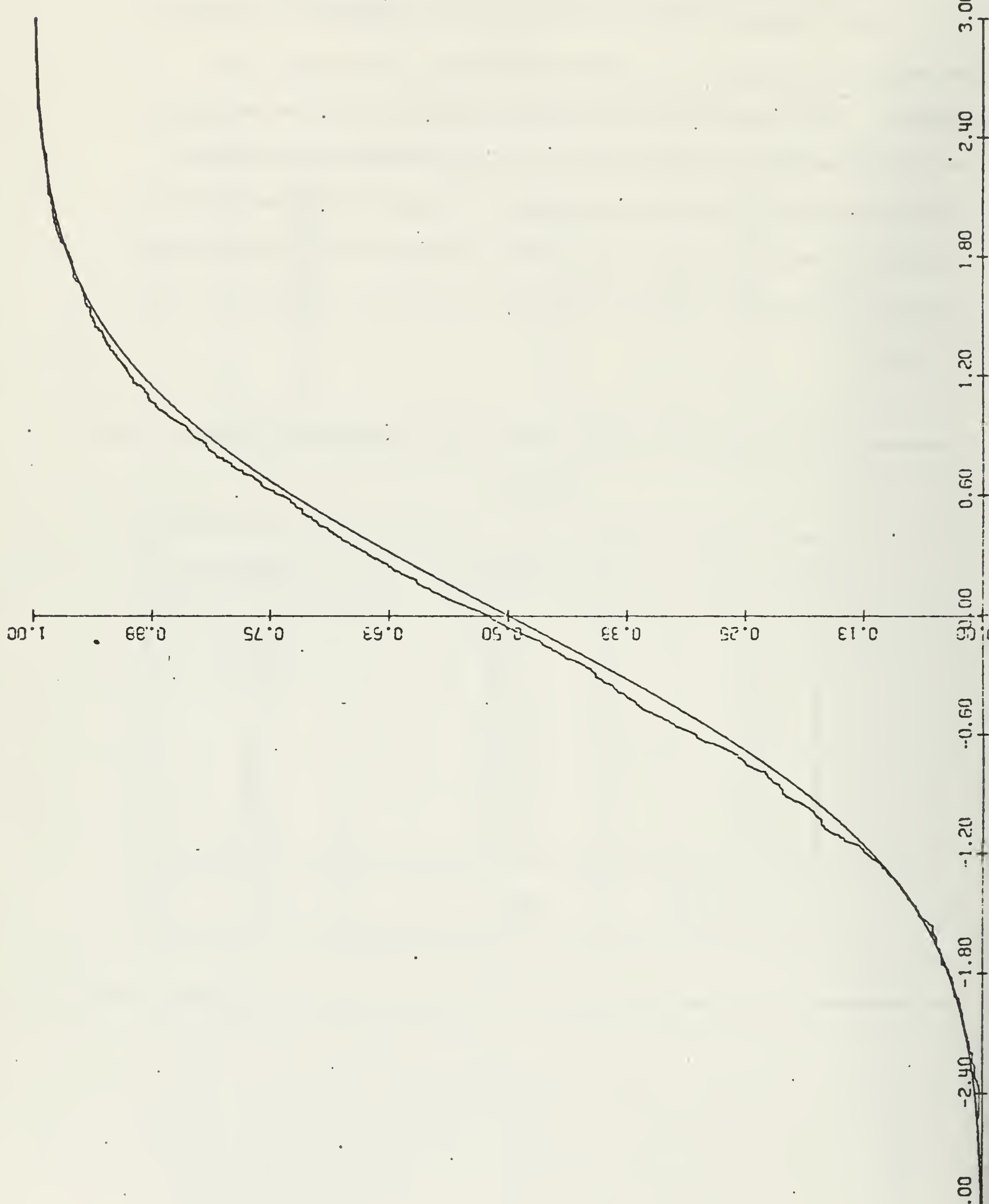
These results, like those of the preceding section are quite acceptable. Overall then, the independence properties of GGNRF are very satisfactory.

D.4 TESTING FOR NORMALITY

A final series of tests was undertaken to examine the suitability of GGNRF in an application calling for normally distributed random numbers. First, the Kolmogorov-Smirnov goodness of fit test was applied to the ten sequences of 1024 numbers with the results shown in Table 5 Following the same format as in previous sections, the sample statistics and their P-levels are given along with the summary chi-square statistic. The plot of the sample distribution function with the worst fit is shown in Figure 5.

TABLE 5

	Statistic	P-level
1A	1.210	.10
1B	.799	.55
2A	.704	.70
2B	.578	.89
3A	1.111	.17
3B	1.187	.12
4A	1.095	.18
4B	.590	.88
5A	1.053	.22
5B	.609	.85
$-2*\Sigma \ln(P_i)$	21.57	.36


-3.00
-2.40
-1.80
-1.20
-0.60
0.00
0.60
1.20
1.80
2.40
3.00
0.13
0.25
0.38
0.50
0.63
0.75
0.88
1.00

Although these results indicate good fit, especially given the relatively large sample size, the sample variances and means were also checked. Given the standard normality hypothesis, the sample variances multiplied by 1024 should be chi-square distributed with 1023 degrees of freedom. The sample variances and the corresponding P-levels under this hypothesis are listed below in Table 6 along with the usual summary statistic..

TABLE 6

	Statistic	P-level
1A	.979	.77
1B	.915	.974
2A	1.106	.009
2B	.936	.925
3A	.997	.51
3B	1.038	.19
4A	1.033	.22
4B	1.022	.30
5A	.963	.79
5B	.998	.50
$-2*\Sigma \ln(P_i)$	22.11	.33

The sample means should be normal with mean 0 and variance 1/1024. Multiplying the sample means by a factor of 32 should result in numbers drawn from a standard normal population. However, the large size of the P-levels in Table 7 gives cause for suspicion that the hypothesis is not true. On the other hand, the sample size of 1024 was originally chosen to be large enough to signal even acceptably small deviations from ideal behavior; the very worst sample mean was only -.062. Furthermore, given the amount of testing undertaken in this study, it is to be expected that sooner or later some test results will go awry. To shed more light on the matter, further tests of the mean tendency of GGNRF seemed appropriate. Since only one seed would be needed for the actual Monte Carlo application, seed #3 was selected for a more intensive examination. Eleven thousand numbers were drawn from seed #3 and then discarded in order to skip over the two strings of 1024 numbers previously tested. Then ten consecutive strings of 1024 numbers were drawn and their sample means were computed. The results are listed in Table 8. In addition, ten new seeds were selected and samples of 1024 numbers drawn from each. The data for these sequences is shown in Table 9.

TABLE 7

	Statistic	P-level
1A	-.049	.94
1B	-.019	.72
2A	.0083	.40
2B	.0069	.41
3A	.034	.14
3B	-.042	.91
4A	-.067	.98
4B	-.021	.75
5A	-.056	.95
5B	-.019	.72
$-2^*\Sigma\ l_n(P_i)$	9.89	.97

TABLE 8

	Statistic	P-level
1	.035	.13
2	.013	.34
3	-.029	.82
4	-.0035	.54
5	-.036	.87
6	-.016	.70
7	-.0054	.57
8	.025	.21
9	.064	.02
10	-.0060	.58
$-2*\Sigma\ \ln(P_i)$	21.925	.34

TABLE 9

	Statistic	P-level
1	.041	.10
2	.0053	.43
3	-.04	.90
4	.023	.23
5	-.010	.63
6	-.024	.78
7	.029	.17
8	.018	.28
9	.0026	.47
10	-.0035	.54
$-2*\Sigma\ \ln(P_i)$	19.74	.48

The results of these tests certainly do much to allay the suspicions raised by the results in Table 7. Neither would it seem that seed #3 has run into an area of systematically bad behavior (witness Table 8), nor would it seem that there is an overall bias in the generator (witness Table 9). These tests together with the preceding K-S tests and sample variance tests indicate that GGNRF has satisfactory distributional properties. Overall then, GGNRF tests out as a satisfactory normal random number generator.

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